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PROJECT WISH: THE EMERALD CITY

PHASE II

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ABSTRACT

The purpose of the Permanently Manned Autonomous Space Oasis, designated Project WISH: The Emerald City, is to serve as permanent living quarters for space colonists. In addition, it will serve as a stopover for space missions and will be capable of restationing itself practically anywhere within the solar system to provide support for these missions. The station should be self-sufficient, with no specific dependence on any resources from Earth.

The 1990-1991 design team continued work started by last year's class. This year, further studies were conducted in the areas of orbital mechanics, propulsion, attitude control, and human factors. Critical elements were identified in each of these areas, and guidelines were established for the design of the Emerald City. Using the knowledge gained from these studies, two particular missions of interest, a Saturn Envelope mission and an Earth-to-Mars mission, were examined. The size and mass estimates, along with the methodologies used in their determination, are considered to be the main accomplishments of Phase II.

ACKNOWLEDGEMENTS

We would like to take this opportunity to thank everyone that has given us assistance during the course of the year. Special thanks go out to Lisa Kohout and Karl Faymon, our mentors from NASA Lewis Research Center, and to Kurt Hack, John Riehl, and Dave Bents from NASA Lewis for coming to Columbus to present seminars. Also, we would like to thank Ohio State students Yong Liu and Patrick Plaisted for their assistance with the computer programs.

Finally, we would like to mention the students who took part in the design class during the winter quarter. These people were Tim Chu, Doug Gulick, Patrick Plaisted, Keith Rabe, Richard Taylor, and Cathy Trenta.

FOREWORD

This report reflects the result of our design classes through the second year of Project WISH. The design activity was carried out through a sequence of three courses offered during the autumn, winter, and spring quarters. The autumn quarter course was a technical elective, two credit-hour, introductory course, mainly dedicated to the understanding of the accomplishments of the first phase (1989-1990) of the project and building the background to be able to further the design. The winter quarter course fulfilled a four credit-hour design course requirement for the seniors. During the autumn and winter quarters, seminars by NASA Lewis personnel were included. The spring quarter class was a three credit-hour technical elective course which culminated in this report. Project WISH is measuring up to our expectations in every regard both in technical and educational aspects.

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NOMENCLATURE

Greek

β - thrust angle
γ - true anomaly
Δ - change in
θ - orientation from inertial reference axes
μ - gravitational parameter, 1.3271544 x 10^{11} km^3/s^2
π - 3.141592653
ρ - density
σ - stress
τ - period, non-dimensional time
Ω - orbital angular velocity

Latin

D - diameter
F - force
I - impulse, moment of inertia
J - impulse
N - number of crewpeople
P - pressure, power
R - ratio of thrust to gravity force, major radius of torus
TOF - time-of-flight
V - velocity, volume
X - inertial reference axis
Y - inertial reference axis
a - acceleration
g - gravitational constant on surface of Earth, 9.81 m/s^2
h - height, orbital angular momentum
m - mass
m - mass flow rate
n - number of engines, torus spin rate
r - radius
t - time, thickness
x - state vector

Subscripts

RMS - root-mean-square
T - total
atm - atmosphere
c - cavity
con - for control
dry - dry (without propellant)
f - final state
ff - free-flight
i - initial, individual
l - payload

Subscripts (continued)

max - maximum
min - minor
p - propellant
pr1 - first powered flight
pr2 - second powered flight
sp - specific
spe - engine specific
syn - synodic
th - thrust
u - uranium
w - engine
0 - starting state
1 - first state
2 - second state

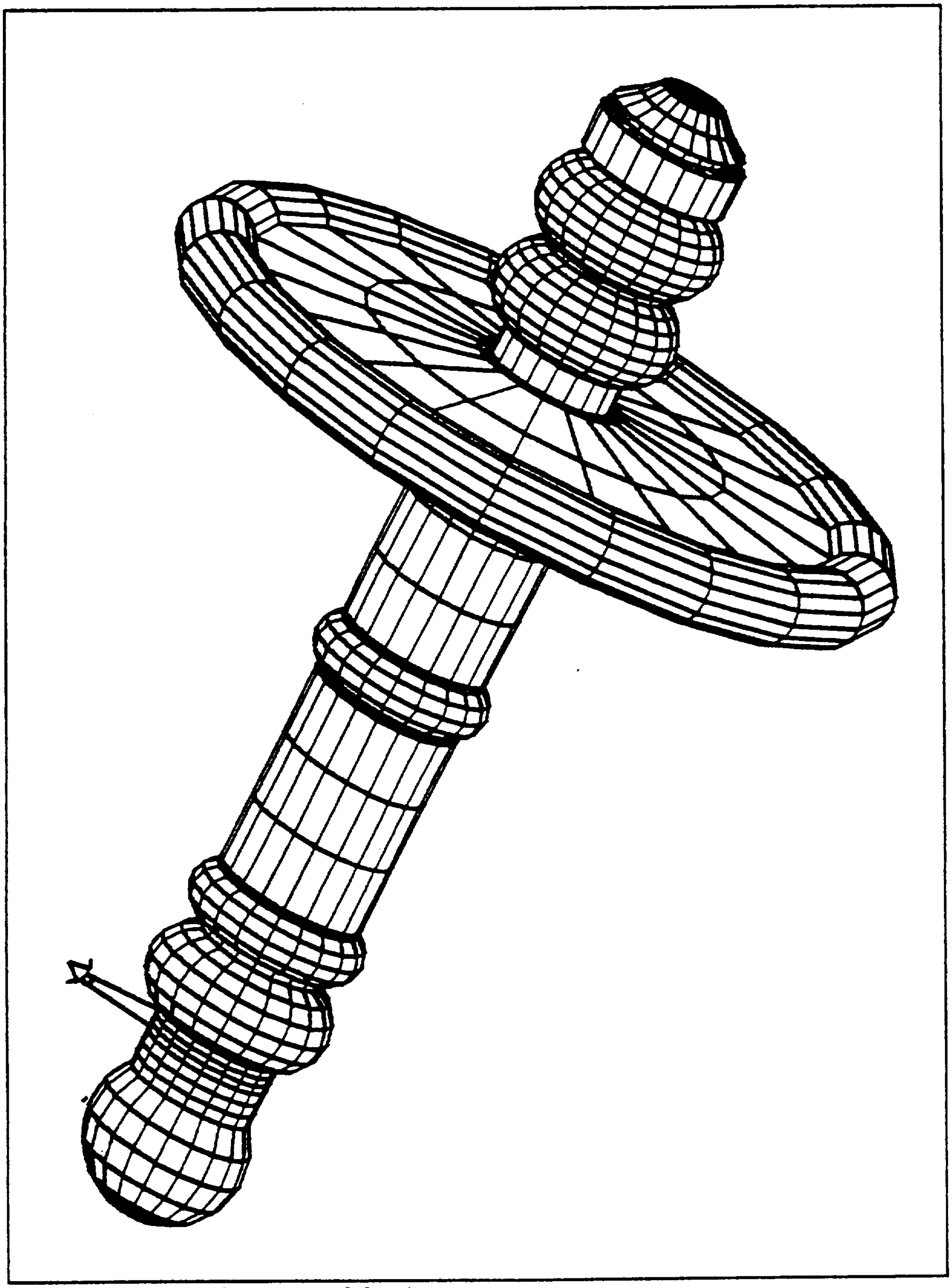

Figure 1.1 - The Emerald City

CHAPTER 1

INTRODUCTION

1.0 PROJECT OVERVIEW

The year is 1620. Winter has struck with a vengeance on the Northeastern coast of the New World. The one hundred and two Pilgrims that have made the journey across the great expanse of the Atlantic Ocean are fighting for their lives. Because there was not time to establish themselves firmly on the shore, the Pilgrims are forced to take shelter on the Mayflower, the ship that brought them across the ocean, for survival. Although some will be lost, those that endure through the first harsh winter will do so because of the support of their ship. With this support, so critical during the initial months of the settlement, the Pilgrims will survive until the spring and establish a permanent colony in the unexplored land of North America.

The year is now 2060. The first manned mission is arriving at the great ringed planet, Saturn. The purpose of this mission is to investigate the Saturnian moon Titan, an object of man's curiosity ever since the Cassini probe first explored it back at the turn of the century. It appears that the speculation about the possibility of colonizing Titan, which started as far back as the 1990's[1], will finally be either proved conclusively or scrapped with the other concepts that never fulfilled their potential. Limitations of the interplanetary ship carrying the expedition have made cargo space precious. Excess amounts of food and supplies have been sacrificed

to accommodate the extensive scientific and data gathering equipment needed to thoroughly explore Titan and the other diverse moons of Saturn. Because of this, the Emerald City has taken up station outside of Saturn's sphere of influence to provide assistance for this historic mission. It will furnish food and equipment for the astronauts, as well as a place for them to take refuge from the desolate vacuum of space. It can also furnish an environment simulating that of Earth, providing artificial gravity as well as the psychological benefits of a large group of people, points which their own ship could not address. Because of the presence of the Emerald City, the mission can remain at Saturn for a much longer period of time. With the extra data that could be collected and recorded, future missions to the moons of Saturn may be made with more confidence, perhaps with the intent of establishing a permanent settlement on Titan.

What is the connection between these two events, separated by over 400 years? The success of both of these scenarios is dependent upon the support provided by the ships in order to make it through the difficult initial stages of their missions. Without this initial support, both missions would be in serious jeopardy. This discussion of past and future events sets the stage for the introduction of Project WISH.

1.1 PROJECT DESCRIPTION

Project WISH (Wandering Interplanetary Space Harbor) is a three year design project currently being conducted at The Ohio

State University. This project deals with the design of a Permanently Manned Autonomous Space Oasis, designated Emerald City (see Figure 1.1). The purpose of Project WISH and the Emerald City is to serve as permanent living quarters for between 500-1000 space inhabitants and to provide crucial support for interplanetary missions in the solar system. To provide this support, the Emerald City will have the ability to relocate itself virtually anywhere in the solar system. This relocation should be accomplished in periods of three years or less whenever possible, thus making the Emerald City flexible in its mission planning. It also should have a lifetime of fifty years, making it an essential link in the space transportation network throughout the latter half of the 21^{st} century.

During the 1990-1991 acedemic year, Phase II of Project WISH was carried out. Phase I, conducted during the 1989-1990 academic year, was a general level study of the major systems required for the Emerald City. During Phase II, a more in-depth study was conducted into the disciplines of orbital mechanics, propulsion, attitude control, and human factors. Critical elements were identified in all of these areas, and guidelines were established for the design of the Emerald City. These guidelines were then combined to carry out the design of two particular missions of interest, a Saturn Envelope mission and an Earth-to-Mars mission. The size and mass estimates of the Emerald City for these missions, along with the methods used to obtain them, are considered the main accomplishments of Phase II, and are the focus of this report.

CHAPTER 2

ORBITAL MECHANICS

2.0 INTRODUCTION

Orbital mechanics is a central discipline in space design, and was one of the areas studied in-depth during Phase II of Project WISH. In general, orbital mechanics deals with the motion of a smaller body, broadly termed as the "satellite", in the gravitational field of a larger body, known as the "central body". In the context of Project WISH, this deals with the motion of the Emerald City in the gravitational field of the sun. Orbital mechanics is used to find the velocity changes, or ΔV's, that are needed for the Emerald City to make a transfer from one place in the solar system to another in a certain time. Last year, some preliminary mission analysis was conducted in an ad hoc manner as an understanding of the computer programs used in the investigation was developed[2]. A preliminary nominal orbit was chosen at 3.2 AU's, but this value was selected due more to the proximity of the asteroid belt than a detailed study of orbital mechanics. Using the experience gained during last year's design effort, a more rigorous study was performed on the transfers for the Emerald City, with the hope that a more substantiated nominal orbit could be chosen. This nominal orbit should yield the most advantageous transfers to as many of the planets as possible. After all, the W in WISH stands for "Wandering", and that is what the Emerald City will be doing throughout the solar system.

2.1 NOMINAL ORBIT

2.1.1 Background

One of the main goals set down for Project WISH this year was the determination of a nominal orbit in heliocentric space. This nominal orbit would act as a sort of "home base" for the Emerald City, a place where it could return between missions for rest and refit. The original design requirement for the project called for a maximum flight time of three years from this chosen orbit to any planet in the solar system. The idea was to make the Emerald City mobile enough to be able to react to orders requiring its presence at a certain planet within a reasonable time frame. However, it was quickly discovered that this requirement placed an untenable burden on the prospective propulsion systems of the Emerald City, even extrapolating well into the 21st century. Thus the design goals were pared down (increasing the flight times to some planets while eliminating trips to other planets altogether) in order to make them more attainable for the time frame being considered.

In the selection of the nominal orbit, an attempt was made to keep the flight times to a minimum while retaining as much of the solar system as possible in the "flight envelope". After much discussion, a new set of design goals emerged for the orbital mechanics and propulsion groups of Project WISH. First of all, it is assumed that the Emerald City will become operational sometime in the mid-21st century. At this time, it is envisioned that humans have become firmly established on the Moon and Mars, and are beginning to look beyond. The next logical step could perhaps be human

exploration of the asteroid belt and the moons of Jupiter. Using such a scenario as background, all of the planets in the solar system were categorized by their perceived importance to human-kind's exploration of the solar system in the mid-21st century. These categories were employed as a guide in the decision-making process of what the nominal orbit should be and how much thrust the propulsion system of the Emerald City would need to provide.

The first category is made up of the planets considered very important and consists of Earth, Mars, and Jupiter. These planets would more than likely be the focus of the space exploration effort during the next century. As mentioned earlier, colonies should already be established on the Moon and Mars, and intensive scientific study with preliminary exploration could be taking place on Jupiter and its moons. Thus, the Emerald City's ability to reach them in three years or less would be essential to provide support to these colonies and/or exploration missions. The second category is planets deemed important and is made up of Venus, Saturn, and Uranus. These are planets that would be targets for scientific study, but are not considered as crucial as the planets in the first category. Thus, it was decided that for these planets, transfers of up to five years would be allowable. The third and final category is planets considered to be of least interest, and includes Mercury, Neptune, and Pluto. While these planets have much to offer in terms of scientific value, it was felt that in the time frame of Project WISH, there would be little need for trips to these planets by a spacecraft like the Emerald City. Thus, the

only trips made to these planets would be in special cases when the planets would lie in a favorable orientation with the Emerald City.

By using the above criteria, the design goals given at the beginning of last year were adjusted to more reasonable levels. Instead of requiring the Emerald City to make transfers from the nominal orbit to any planet 100% of the time in three years, this requirement was only applied to the planets in the very important category (Earth, Mars, Jupiter). For the planets in the important category (Venus, Saturn, Uranus), these requirements were eased somewhat by allowing transfers up to five years if necessary, and by not requiring the Emerald City to make the transfer 100% of the time. The design goal would instead be that the Emerald City should be able to make the transfer as much of the time as possible and still keep the propulsion system requirements somewhat reasonable. For the third category (Mercury, Neptune, Pluto), there were no specific design goals set; any transfers that would be possible in less than five years would be considered icing on the cake. Thus, with the above criteria established, the mission analysis was carried out.

2.1.2 Mission Analysis

The main difficulty in the mission analysis portion of Project WISH is the lack of a particular mission to analyze. By definition, the Emerald City is to have the capability to travel wherever it may be needed in the solar system within a reasonable time period. For example, if some crucial product is required for

a fledgling colony on a Jovian moon, the Emerald City will not have the luxury of waiting years for the opening of a launch window to Jupiter before leaving the nominal orbit. Instead, it must be able to make the transfer immediately, regardless of where the planet might be with respect to the Emerald City. This so-called "transfer-on-demand" results in high ΔV's, which is the amount of velocity change that must be provided to the Emerald City in order for it to achieve a transfer from the nominal orbit to a planet. In turn, this high ΔV places a considerable burden on the propulsion system of Project WISH. Thus, it is hoped to keep these mission ΔV's down to reasonable values in order to alleviate some of the propulsion problems associated with large velocity changes (see Chapter 3).

Hence, the question arises as to how to perform the mission analysis for the Emerald City. An ad hoc approach of picking target planets and launch dates for sample cases was discussed, but was ruled out due to its lack of rigor and tedious nature. Instead, a methodology was established that allowed a more thorough investigation of the transfers between a nominal orbit and the planets in the solar system to be performed. This methodology is based on a concept known as the synodic period between two orbiting bodies (see Figure 2.1). The synodic period is the time it takes for a certain angular orientation known as the phase angle between two bodies to repeat itself. For example, as shown in Figure 2.1, at some time t = 0, the Emerald City and the target planet will lie in a straight line connecting the two to the sun. At this point, the

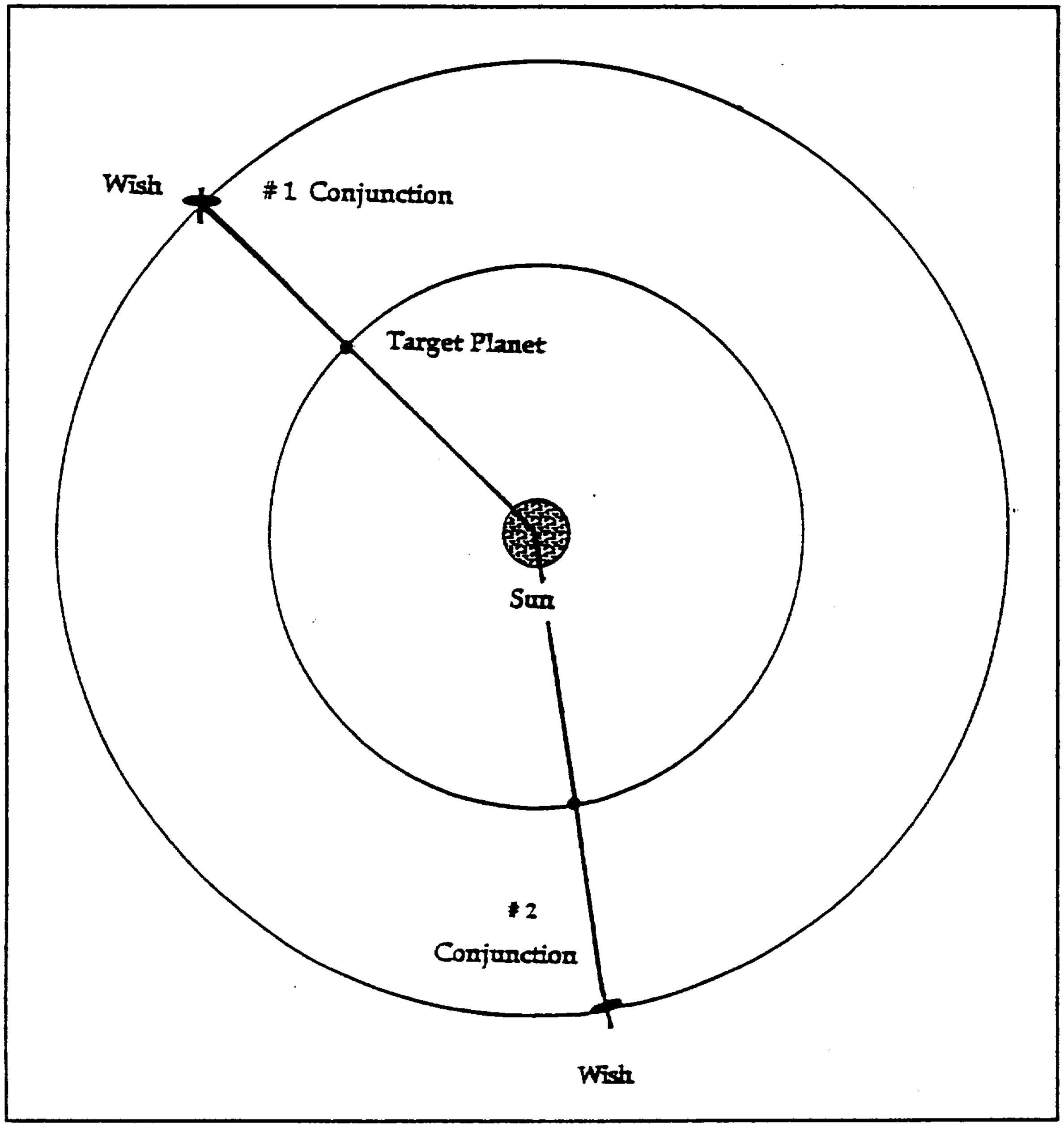

Figure 2.1 - The Synodic Period

phase angle between the two is zero, which is known as conjunction. However, the Emerald City will be travelling slower than the target planet due to the fact that it lies further from the sun. Thus, at a future time, the target planet will essentially "lap" the Emerald City and they will once again be in conjunction at some new posi-

tion in their orbits. The time it takes for this to occur is the called the synodic period, and is denoted by τ_{syn}. The synodic period is related to the angular velocities of the two bodies, Ω_{EC} and Ω_{TO}, and is given by

$$\tau_{syn} = \frac{2\pi}{|\Omega_{TO} - \Omega_{EC}|}. \qquad (2.1)$$

Since the synodic period represents the time it takes for a phase angle to repeat itself, it can be extremely helpful in the mission analysis. Because the ΔV required to make a heliocentric transfer between two bodies in a given time is a function of the orientation of the bodies, it will be nearly cyclic in time, with a period that is given by the synodic period. Thus, instead of looking at transfers throughout the entire fifty year time period that the Emerald City is envisioned to be operating in, they can be examined over a single synodic period. This will result in values for ΔV that can be applied to the transfer at any future time. There are two assumptions on the geometry of the orbits that were made in the statement that the ΔV was cyclic with time. The first assumption is that the orbits of the planets are circular instead of elliptical, while the second assumption is that the nominal and planetary orbits all lie in the same plane. If these assumptions were invalid, then the ΔV would not be cyclic with time as was stated, but would depend on where the planet is located in its orbit. It can be seen from Table 2.1[3] that, while these assumptions are not perfect, they are adequate for this preliminary investigation for all of the planets except Mercury and Pluto.

Table 2.1 - Eccentricity and Inclination of the Planets

Planet	Eccentricity	Inclination
Mercury	0.21	7.0^0
Venus	0.01	3.4^0
Earth	0.02	0.0^0
Mars	0.09	1.9^0
Jupiter	0.05	1.3^0
Saturn	0.05	2.5^0
Uranus	0.05	0.8^0
Neptune	0.01	1.8^0
Pluto	0.26	17.1^0

Eccentricity = 0 ==> circular orbit

Inclination = 0^0 ==> same orbital plane as Earth

Using the computer program MULIMP[4] obtained from NASA Lewis Research Center and a simple code written to calculate the synodic periods for a given nominal orbit radius, the mission analysis for Project WISH was conducted. MULIMP is a FORTRAN code that calculates the ΔV required for interplanetary transfers under the assumption that the change in velocity takes place instantaneously, i.e. infinite acceleration. Again, this assumption should not hamper the study because the propulsion system powered-flight times should be sufficiently short with respect to the overall flight time to keep this assumption valid for this precursory investigation. The steps in the mission analysis then proceeded as follows. First, a nominal orbit radius was chosen and input into

the code which calculated the synodic periods for this case. Then, MULIMP was run for each planet at many different launch dates (typically between 100 and 200) over a synodic period and for flight times of one to five years in order to obtain the relationship between the ΔV, the flight time, and the fraction of time into the synodic period that the launch occurred at. This resulted in graphs similar to the one shown in Figure 2.2. This particular graph is for a transfer from a nominal orbit of 3 AU's (astronomical units, 1 AU is distance from Earth to Sun) to Jupiter. It can be seen how the ΔV varies over the synodic period; at some points it is relatively low while at other points, it is quite high. It can also be seen that at the completion of one synodic period, the ΔV is nearly the same as it was at the beginning, as expected. The small difference can be attributed to the two assumptions that were made about the geometries of the orbits. While these assumptions were made to facilitate the analysis, MULIMP uses the actual orbits of the planets, complete with eccentricity and inclination. Thus, the ΔV at each of the endpoints of the synodic period does not match exactly, but the fact that they are so close is a good indication of the validity of the assumptions.

To plot out and analyze graphs such as Figure 2.2 for all of the planets at each nominal orbit considered would be quite tedious. Then, the question would be what would one do with the piles of graphs that had been generated. To aid in the analysis of the data obtained from MULIMP, a program was written that would

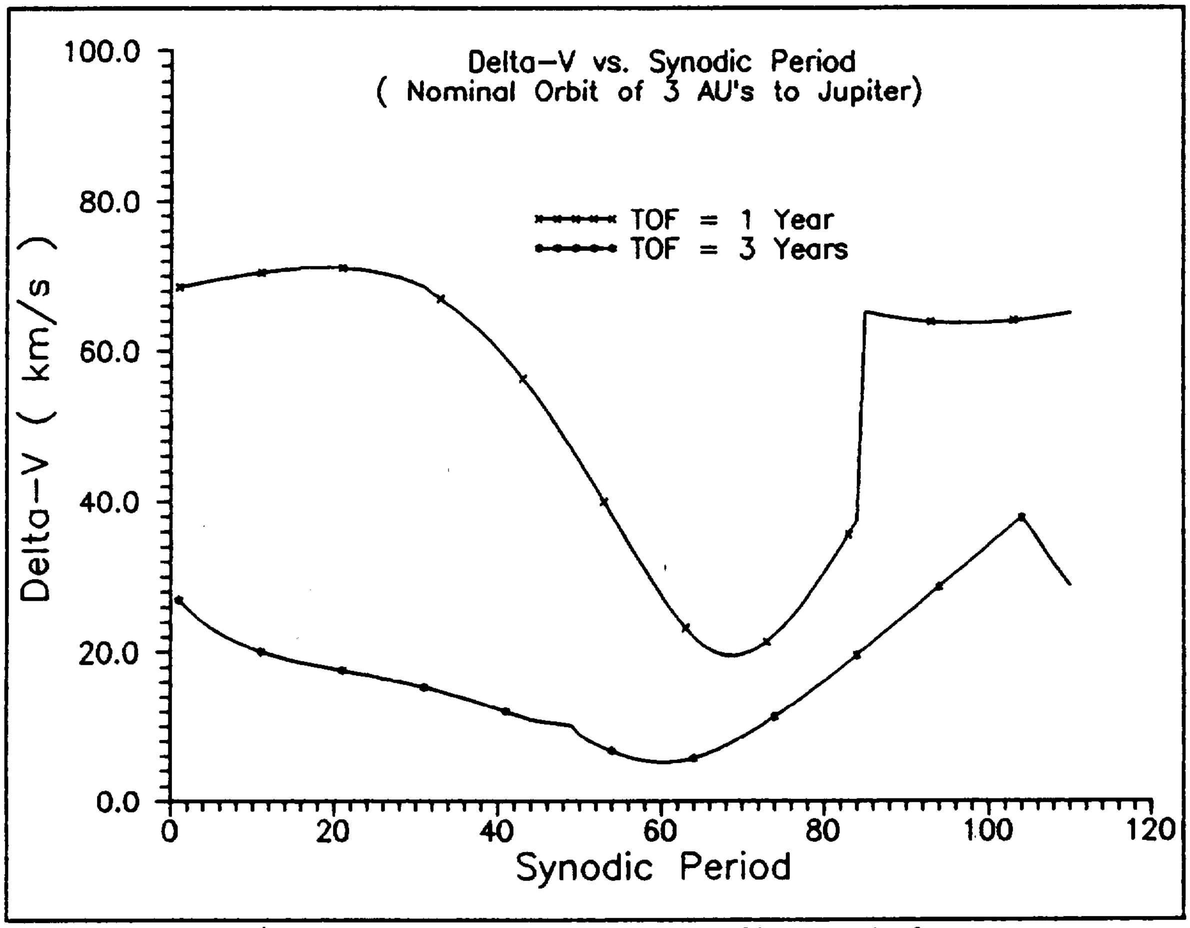

Figure 2.2 - ΔV Behavior Over One Synodic Period (Flight Time is a Parameter)

calculate the percentage of time during the synodic period that a transfer from a nominal orbit to a planet would fall beneath a certain value of ΔV. Then, this data could be presented in the form of a chart, as shown in Table 2.2. The left-hand side of the chart shows what nominal orbit was used, to what planet the transfer was made, and what was the flight time used. For example, in Table 2.2, N3-P5-T3 would mean a transfer from a nominal orbit of 3 AU's to Jupiter (planet #5), with a flight time of three years. The numbers on the right of the chart represent the percentage of time during a synodic period that a transfer falls

Table 2.2 - Percentages When Transfer is Possible Given ΔV Limit
(Middle Seven Planets, Nominal Orbit = 3 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N3-P2-T1	0.0	16.5	38.8	56.5	71.4	85.1	97.6	100
N3-P2-T2	0.0	9.4	33.3	49.8	64.7	78.0	91.4	100
N3-P2-T3	0.0	0.0	26.3	44.3	59.2	73.7	87.5	100
N3-P2-T4	0.0	0.0	20.4	40.0	56.1	70.6	84.3	98.8
N3-P2-T5	0.0	0.0	18.0	36.5	52.9	67.8	82.4	97.6
OPTIMAL FLIGHT TIME = 1.122								
N3-P3-T1	0.0	25.6	50.2	70.0	85.0	98.2	100	100
N3-P3-T2	0.0	28.6	48.0	64.3	78.0	91.6	100	100
N3-P3-T3	0.0	18.1	40.5	57.3	72.7	86.8	100	100
N3-P3-T4	0.0	8.8	34.8	52.9	68.3	83.7	99.1	100
N3-P3-T5	0.0	0.0	31.3	49.3	65.6	81.5	96.9	100
OPTIMAL FLIGHT TIME = 1.394								
N3-P4-T1	0.0	31.5	57.0	85.6	96.3	100	100	100
N3-P4-T2	13.7	44.1	60.7	74.1	87.8	100	100	100
N3-P4-T3	9.3	43.0	61.9	75.6	86.7	98.5	100	100
N3-P4-T4	0.0	28.1	48.9	63.7	78.5	96.7	100	100
N3-P4-T5	0.0	29.3	52.6	68.9	81.5	93.7	100	100
OPTIMAL FLIGHT TIME = 1.764								
N3-P5-T1	0.0	7.3	20.9	29.1	34.5	40.9	92.7	100
N3-P5-T2	11.8	32.7	83.6	99.1	100	100	100	100
N3-P5-T3	24.5	70.0	89.1	100	100	100	100	100
N3-P5-T4	39.1	70.0	87.3	100	100	100	100	100
N3-P5-T5	40.9	68.2	84.5	100	100	100	100	100
OPTIMAL FLIGHT TIME = 4.466								
N3-P6-T1	0.0	0.0	0.0	0.0	0.0	0.0	18.7	29.0
N3-P6-T2	0.0	0.0	8.8	34.7	49.2	100	100	100
N3-P6-T3	0.0	18.1	45.6	94.3	100	100	100	100
N3-P6-T4	0.0	37.8	81.9	100	100	100	100	100
N3-P6-T5	11.9	57.0	87.0	100	100	100	100	100
OPTIMAL FLIGHT TIME = 6.860								
N3-P7-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P7-T2	0.0	0.0	0.0	0.0	0.0	0.0	13.8	33.5
N3-P7-T3	0.0	0.0	0.0	0.0	27.2	48.8	100	100
N3-P7-T4	0.0	0.0	0.0	35.4	75.2	100	100	100
N3-P7-T5	0.0	0.0	29.9	69.7	100	100	100	100
OPTIMAL FLIGHT TIME = 12.960								
N3-P8-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P8-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P8-T3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	13.7
N3-P8-T4	0.0	0.0	0.0	0.0	0.0	18.8	42.6	91.9
N3-P8-T5	0.0	0.0	0.0	0.0	26.9	54.3	100	100

below a certain ΔV. By using charts such as the one found in Table 2.2, different transfer characteristics can be examined quickly and completely, as opposed to a piecemeal approach, which would be time consuming and not as thorough. Thus, now the merits of different nominal orbits can be compared.

2.1.3 Analysis Results

The resulting charts from the mission analysis can be found in Tables 2.3-2.10 for nominal orbits of 3, 4, 5, and 10 AU's. These were broken down into the inner planets in Tables 2.3-2.6 and the outer planets in Tables 2.7-2.10.

Table 2.3 - Percentages When Flight is Possible Given ΔV Limit (Inner Planets, Nominal Orbit = 3 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N3-P1-T1	0.0	0.0	15.1	31.2	45.2	61.3	76.3	90.3
N3-P1-T2	0.0	0.0	8.6	25.8	39.8	54.8	69.9	86.0
N3-P1-T3	0.0	0.0	3.2	21.5	34.4	50.5	65.6	80.6
N3-P1-T4	0.0	0.0	0.0	18.3	32.3	46.2	62.4	78.5
N3-P1-T5	0.0	0.0	0.0	15.1	29.0	44.1	58.1	76.3
N3-P2-T1	0.0	16.5	38.8	56.5	71.4	85.1	97.6	100
N3-P2-T2	0.0	9.4	33.3	49.8	64.7	78.0	91.4	100
N3-P2-T3	0.0	0.0	26.3	44.3	59.2	73.7	87.5	100
N3-P2-T4	0.0	0.0	20.4	40.0	56.1	70.6	84.3	98.8
N3-P2-T5	0.0	0.0	18.0	36.5	52.9	67.8	82.4	97.6
N3-P3-T1	0.0	25.6	50.2	70.0	85.0	98.2	100	100
N3-P3-T2	0.0	28.6	48.0	64.3	78.0	91.6	100	100
N3-P3-T3	0.0	18.1	40.5	57.3	72.7	86.8	100	100
N3-P3-T4	0.0	8.8	34.8	52.9	68.3	83.7	99.1	100
N3-P3-T5	0.0	0.0	31.3	49.3	65.6	81.5	96.9	100
N3-P4-T1	0.0	31.5	57.0	85.6	96.3	100	100	100
N3-P4-T2	13.7	44.1	60.7	74.1	87.8	100	100	100
N3-P4-T3	9.3	43.0	61.9	75.6	86.7	98.5	100	100
N3-P4-T4	0.0	28.1	48.9	63.7	78.5	96.7	100	100
N3-P4-T5	0.0	29.3	52.6	68.9	81.5	93.7	100	100

Table 2.4 - Percentages When Flight is Possible Given ΔV Limit (Inner Planets, Nominal Orbit = 4 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N4-P1-T1	0.0	0.0	0.0	15.2	33.7	51.1	63.0	73.9
N4-P1-T2	0.0	0.0	0.0	27.2	43.5	56.5	66.3	76.1
N4-P1-T3	0.0	0.0	0.0	19.6	39.1	52.2	64.1	73.9
N4-P1-T4	0.0	0.0	0.0	10.9	35.9	48.9	62.0	71.7
N4-P1-T5	0.0	0.0	0.0	8.7	31.5	46.7	59.8	69.6
N4-P2-T1	0.0	0.0	18.3	41.5	61.0	79.3	95.1	100
N4-P2-T2	0.0	12.2	35.4	53.7	68.3	82.9	96.3	100
N4-P2-T3	0.0	7.3	31.7	51.2	64.6	79.3	93.9	100
N4-P2-T4	0.0	0.0	25.6	45.1	59.8	74.4	89.0	100
N4-P2-T5	0.0	0.0	19.5	41.5	58.5	74.4	89.0	100
N4-P3-T1	0.0	0.0	27.1	50.7	75.0	93.6	100	100
N4-P3-T2	0.0	29.3	50.0	66.4	81.4	95.7	100	100
N4-P3-T3	0.0	24.3	45.0	61.4	76.4	91.4	100	100
N4-P3-T4	0.0	17.1	40.0	57.1	73.6	87.9	100	100
N4-P3-T5	0.0	10.7	36.4	55.0	70.7	85.7	100	100
N4-P4-T1	0.0	12.4	34.5	60.2	95.6	100	100	100
N4-P4-T2	10.6	41.6	65.5	85.8	100	100	100	100
N4-P4-T3	7.1	40.7	63.7	77.9	95.6	100	100	100
N4-P4-T4	0.0	31.9	54.0	74.3	94.7	100	100	100
N4-P4-T5	0.0	25.7	54.0	73.5	90.3	99.1	100	100

Table 2.5 - Percentages When Flight is Possible Given ΔV Limit
(Inner Planets, Nominal Orbit = 5 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N5-P1-T1	0.0	0.0	0.0	8.8	26.4	37.4	53.8	69.2
N5-P1-T2	0.0	0.0	7.7	23.1	41.8	54.9	69.2	83.5
N5-P1-T3	0.0	0.0	0.0	23.1	40.7	56.0	68.1	81.3
N5-P1-T4	0.0	0.0	0.0	22.0	38.5	51.6	67.0	79.1
N5-P1-T5	0.0	0.0	0.0	17.6	37.4	50.5	64.8	78.0
N5-P2-T1	0.0	0.0	0.0	21.7	40.8	62.5	83.3	100
N5-P2-T2	0.0	2.5	33.3	51.7	68.3	84.2	100	100
N5-P2-T3	0.0	0.8	31.7	50.0	66.7	81.7	97.5	100
N5-P2-T4	0.0	0.0	26.7	46.7	64.2	79.2	95.8	100
N5-P2-T5	0.0	0.0	24.2	44.2	60.8	77.5	93.3	100
N5-P3-T1	0.0	0.0	0.0	28.9	51.1	78.5	100	100
N5-P3-T2	0.0	23.7	46.7	66.7	83.7	99.3	100	100
N5-P3-T3	0.0	25.9	46.7	65.2	80.7	97.0	100	100
N5-P3-T4	0.0	21.5	43.0	62.2	77.8	94.1	100	100
N5-P3-T5	0.0	17.0	40.0	59.3	76.3	91.9	100	100
N5-P4-T1	0.0	0.0	12.2	31.7	52.5	94.2	100	100
N5-P4-T2	0.0	33.8	64.0	84.2	99.3	100	100	100
N5-P4-T3	9.4	41.7	64.0	83.5	100	100	100	100
N5-P4-T4	0.0	37.4	59.7	77.7	97.1	100	100	100
N5-P4-T5	0.0	34.5	56.1	76.3	97.1	100	100	100

Table 2.6 - Percentages When Flight is Possible Given ΔV Limit (Inner Planets, Nominal Orbit = 10 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N10-P1-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	14.4
N10-P1-T2	0.0	0.0	0.0	4.4	22.2	36.7	51.1	68.9
N10-P1-T3	0.0	0.0	5.6	18.9	36.7	53.3	68.9	87.8
N10-P1-T4	0.0	0.0	11.1	28.9	43.3	58.9	76.7	93.3
N10-P1-T5	0.0	0.0	13.3	30.0	45.6	61.1	77.8	94.4
N10-P2-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	17.2
N10-P2-T2	0.0	0.0	0.0	17.2	35.3	56.0	76.7	97.4
N10-P2-T3	0.0	0.0	15.5	37.9	57.8	75.9	96.6	100
N10-P2-T4	0.0	0.0	27.6	48.3	66.4	84.5	100	100
N10-P2-T5	0.0	0.0	31.9	51.7	68.1	85.3	100	100
N10-P3-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	19.7
N10-P3-T2	0.0	0.0	0.0	22.0	43.3	68.5	95.3	100
N10-P3-T3	0.0	0.0	28.3	50.4	73.2	95.3	100	100
N10-P3-T4	0.0	14.2	39.4	61.4	81.1	100	100	100
N10-P3-T5	0.0	21.3	44.1	64.6	84.3	100	100	100
N10-P4-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21.8
N10-P4-T2	0.0	0.0	0.0	25.9	52.4	87.1	100	100
N10-P4-T3	0.0	0.0	34.7	66.7	94.6	100	100	100
N10-P4-T4	0.0	23.8	53.7	79.6	99.3	100	100	100
N10-P4-T5	0.0	32.0	59.9	83.7	100	100	100	100

Table 2.7 - Percentages When Flight is Possible Given ΔV Limit (Outer Planets, Nominal Orbit = 3 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N3-P5-T1	0.0	7.3	20.9	29.1	34.5	40.9	92.7	100
N3-P5-T2	11.8	32.7	83.6	99.1	100	100	100	100
N3-P5-T3	24.5	70.0	89.1	100	100	100	100	100
N3-P5-T4	39.1	70.0	87.3	100	100	100	100	100
N3-P5-T5	40.9	68.2	84.5	100	100	100	100	100
N3-P6-T1	0.0	0.0	0.0	0.0	0.0	0.0	18.7	29.0
N3-P6-T2	0.0	0.0	8.8	34.7	49.2	100	100	100
N3-P6-T3	0.0	18.1	45.6	94.3	100	100	100	100
N3-P6-T4	0.0	37.8	81.9	100	100	100	100	100
N3-P6-T5	11.9	57.0	87.0	100	100	100	100	100
N3-P7-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P7-T2	0.0	0.0	0.0	0.0	0.0	0.0	13.8	33.5
N3-P7-T3	0.0	0.0	0.0	0.0	27.2	48.8	100	100
N3-P7-T4	0.0	0.0	0.0	35.4	75.2	100	100	100
N3-P7-T5	0.0	0.0	29.9	69.7	100	100	100	100
N3-P8-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P8-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P8-T3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	13.7
N3-P8-T4	0.0	0.0	0.0	0.0	0.0	18.8	42.6	91.9
N3-P8-T5	0.0	0.0	0.0	0.0	26.9	54.3	100	100
N3-P9-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P9-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P9-T3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P9-T4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P9-T5	0.0	0.0	0.0	0.0	0.0	0.0	19.1	40.4

Table 2.8 - Percentages When Flight is Possible Given ΔV Limit (Outer Planets, Nominal Orbit = 4 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N4-P5-T1	0.0	11.0	18.5	24.0	29.5	36.3	43.8	94.5
N4-P5-T2	12.3	28.1	50.7	99.3	100	100	100	100
N4-P5-T3	22.6	78.8	95.9	100	100	100	100	100
N4-P5-T4	46.6	79.5	94.5	100	100	100	100	100
N4-P5-T5	56.8	80.1	93.8	100	100	100	100	100
N4-P6-T1	0.0	0.0	0.0	0.0	1.4	16.7	20.8	25.0
N4-P6-T2	0.0	0.0	20.1	32.6	41.7	98.6	100	100
N4-P6-T3	0.0	22.2	45.1	99.3	100	100	100	100
N4-P6-T4	2.1	38.2	94.4	99.3	100	100	100	100
N4-P6-T5	18.8	67.4	96.5	100	100	100	100	100
N4-P7-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N4-P7-T2	0.0	0.0	0.0	0.0	0.0	0.0	20.3	29.7
N4-P7-T3	0.0	0.0	0.0	0.0	28.4	42.6	100	100
N4-P7-T4	0.0	0.0	5.4	36.5	70.9	100	100	100
N4-P7-T5	0.0	0.0	31.1	73.0	100	100	100	100
N4-P8-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N4-P8-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N4-P8-T3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	17.1
N4-P8-T4	0.0	0.0	0.0	0.0	0.0	20.2	41.1	94.6
N4-P8-T5	0.0	0.0	0.0	0.0	27.9	49.6	100	100
N4-P9-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N4-P9-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N4-P9-T3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N4-P9-T4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N4-P9-T5	0.0	0.0	0.0	0.0	0.0	0.0	15.4	41.0

Table 2.9 - Percentages When Flight is Possible Given ΔV Limit (Outer Planets, Nominal Orbit = 5 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N5-P5-T1	6.2	15.8	18.1	22.6	24.3	29.4	35.0	40.7
N5-P5-T2	15.8	26.6	40.7	100	100	100	100	100
N5-P5-T3	20.9	57.1	98.9	100	100	100	100	100
N5-P5-T4	32.8	87.6	99.4	100	100	100	100	100
N5-P5-T5	64.4	84.7	98.9	100	100	100	100	100
N5-P6-T1	0.0	0.0	0.0	0.0	12.9	16.3	19.7	22.4
N5-P6-T2	0.0	4.8	22.4	29.3	36.1	97.3	100	100
N5-P6-T3	0.0	23.8	40.1	98.6	100	100	100	100
N5-P6-T4	11.6	36.7	100	100	100	100	100	100
N5-P6-T5	20.4	73.5	99.3	100	100	100	100	100
N5-P7-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N5-P7-T2	0.0	0.0	0.0	0.0	0.0	0.0	20.3	27.2
N5-P7-T3	0.0	0.0	0.0	0.0	27.8	38.0	100	100
N5-P7-T4	0.0	0.0	8.2	35.4	54.4	100	100	100
N5-P7-T5	0.0	0.0	31.6	65.2	100	100	100	100
N5-P8-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N5-P8-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N5-P8-T3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21.1
N5-P8-T4	0.0	0.0	0.0	0.0	0.0	23.1	38.1	60.5
N5-P8-T5	0.0	0.0	0.0	0.0	29.3	47.6	100	100
N5-P9-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N5-P9-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N5-P9-T3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N5-P9-T4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N5-P9-T5	0.0	0.0	0.0	0.0	0.0	0.0	18.0	39.5

Table 2.10 - Percentages When Flight is Possible Given ΔV Limit
(Outer Planets, Nominal Orbit = 10 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N10-P5-T1	0.0	0.0	0.0	0.0	11.0	14.8	17.4	20.6
N10-P5-T2	0.0	0.0	20.6	27.1	32.9	41.9	100	100
N10-P5-T3	0.0	21.3	36.1	67.1	100	100	100	100
N10-P5-T4	7.7	35.5	94.8	100	100	100	100	100
N10-P5-T5	17.4	54.2	100	100	100	100	100	100
N10-P6-T1	5.2	7.0	7.4	10.0	13.0	13.9	14.3	17.8
N10-P6-T2	6.5	11.3	14.8	19.1	22.2	27.8	33.0	41.7
N10-P6-T3	8.3	20.4	27.0	33.5	44.8	100	100	100
N10-P6-T4	15.7	27.4	35.2	97.0	100	100	100	100
N10-P6-T5	19.1	34.3	97.0	100	100	100	100	100
N10-P7-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N10-P7-T2	0.0	0.0	0.0	0.0	7.8	12.4	15.7	19.0
N10-P7-T3	0.0	0.0	0.0	15.0	19.6	24.8	30.1	36.6
N10-P7-T4	0.0	0.0	16.3	22.9	29.4	38.6	100	100
N10-P7-T5	0.0	10.5	23.5	32.0	87.6	100	100	100
N10-P8-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N10-P8-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N10-P8-T3	0.0	0.0	0.0	0.0	0.0	0.0	15.5	20.8
N10-P8-T4	0.0	0.0	0.0	0.0	15.5	22.6	29.2	36.3
N10-P8-T5	0.0	0.0	0.0	16.7	26.2	34.5	47.6	100
N10-P9-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N10-P9-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N10-P9-T3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N10-P9-T4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0
N10-P9-T5	0.0	0.0	0.0	0.0	0.0	0.0	20.9	29.9

Inner Planets

After reading the above discussion and examining Tables 2.3-2.6, one might be tempted to arrive at a conclusion similar to the following. "Well, since planet Earth fell into the first category of planets that had to be reached 100% of the time in three years or less, and according to Tables 2.3-2.6, Earth can be reached 100% of the time in three years for ΔV's greater than 60 km/s, then the propulsion system will have to provide a one-way ΔV of greater than 60 km/s for the design goals to be met." However, upon closer inspection, this conclusion is incorrect due mainly to two reasons. The first reason is that the synodic period of the inner planets are rather low, as can be seen in Table 2.7. They range from about

Table 2.11 - Synodic Periods of the Inner Planets

Planet -	Mercury	Venus	Earth	Mars
τ_{syn} (years)				
n = 3 AU	.25	.70	1.24	2.95
n = 4 AU	.25	.67	1.14	2.46
n = 5 AU	.25	.65	1.10	2.26

one-quarter of a year for Mercury to over two years for Mars. This is a key point in determining what ΔV's will need to be provided for transfers to these inner planets. Because the synodic period is so short, the Emerald City would pass through the entire period in less than three years time. Thus, the Emerald City would have the luxury of waiting at its nominal orbit for some amount of time

after receiving its orders to move, until the ΔV requirements are sufficiently low to make a transfer possible in less than three years and it would still arrive within three years of the time it received its orders to go somewhere.

For example, say that the Emerald City is in a nominal orbit at 4 AU's from the sun. On February 10, 2051, it receives orders to travel to Earth, and its arrival is required within two years. On February 10, the ΔV required to make a two year transfer at this time is found to be 63.9 km/s, a rather high value that would place a large burden on the propulsion system. So instead, the Emerald City decides to wait at the nominal orbit until July 3, 2051, when the ΔV required to make a one year transfer is only 23.1 km/sec. This value is much more easily obtained by the propulsion system, and the Emerald City arrives at Earth on July 2, 2052, about 222 days ahead of the time it was required to be there. This scenario, calculated using MULIMP, is possible because the synodic period for Earth is only 1.14 years, short enough to allow all of the possible phase angles to present themselves to the Emerald City within the time that it is required for it to make the transfer. Thus, instead of leaving immediately, it can wait for some length of time, and then make a transfer that, even with a shorter flight time, has a lower ΔV because of a more favorable orientation.

The second reason that the percentages that a transfer is possible for a given flight time cannot be taken at face value is because of the following fact. In this analysis, the flight time for each transfer is specified, and the ΔV required to make this

exact transfer is found. One might expect that this ΔV would go down as the flight time is made longer, because of the less demanding duration. In general, however, this is not the case for the inner planets. The optimal flight time (the flight time resulting in the lowest ΔV) for the inner planets is on the order of one to two years. Thus, by forcing a trip to one of these planets in, say, three years, the ΔV required actually increases. This is because the trajectory will not go directly to the target planet, but will stall by taking an indirect route in order for the flight time to be the specified value. This phenomenon can be seen in the following example, predicted by MULIMP. The Emerald City receives orders to proceed to Venus on February 27, 2050, and is ordered to be there within 2.5 years. At this point, if the Emerald City were to try to make a transfer with a flight time of 2.5 years, the ΔV required would be 46.1 km/s. However, by making the transfer in only 2.33 years, the ΔV would only be 18.8 km/s, a much better result.

Thus, because of the nature of the transfers to the inner planets for the two reasons cited above, the actual percentages of time that a flight is possible for a given ΔV is rendered meaningless. Instead, what should be looked at is the lowest ΔV a transfer can be made at all with a flight time of three years minus the synodic period. If a transfer can be made at any point in the synodic period for this flight time, then the transfer can be made 100% of the time. Again using the Earth example, in 1.14 years, all of the phase angles between it and the Emerald City will have

occurred due to the length of the synodic period. Thus, if Earth can be reached any percent of the time in 1.86 years or less with a given ΔV, then it can be said that the transfer can be made 100% of the time in under three years from receiving its orders, because it can wait until the favorable orientation appears.

Outer Planets

The results from the mission analysis for the outer planets (Jupiter to Pluto) can be found in Tables 2.7-2.10 for nominal orbits of 3, 4, 5, and 10 AU's. The investigation of the transfers to the outer planets must be treated differently than to the inner planets because the synodic periods are much longer for the outer planets, as seen in Table 2.12. Because of this, the Emerald City

Table 2.12 - Synodic Periods of the Outer Planets

Planet -	Jupiter	Saturn	Uranus	Neptune	Pluto
τ_{syn} (years)					
n = 3 AU	9.26	6.31	5.54	5.37	5.31
n = 4 AU	24.62	10.98	8.85	8.41	8.27
n = 5 AU	196.66	18.02	12.91	12.00	11.71

will encounter only a fraction of the possible phase angles, and must be prepared to make transfers at less than optimum conditions. Thus, for the outer planets, the percentage of time that a transfer is possible for a fixed ΔV becomes important, a yardstick that different nominal orbits can be measured by.

It should be noted what effect moving the nominal orbit further from the sun had on the ability of the Emerald City to reach the outer planets. As seen in Tables 2.7-2.10, there was little benefit gained by moving the nominal orbit away from the sun, and in some cases, the percentages actually went down. This was an important consideration for the selection of the nominal orbit.

2.1.4 Nominal Orbit Selection

The sample nominal orbits that were examined were at 3, 4, 5, and 10 AU's. After examining the percentage tables generated by MULIMP and some discussion, an orbit at 4 AU's was selected as the preliminary nominal orbit. This selection was based on several factors. The first one is the central location of this orbit between Mars, at 1.5 AU and Jupiter, at 5.2 AU. This orbit puts the Emerald City relatively near to the asteroid belt, where vast deposits of valuable materials can be found, and close to the presumed focal points of interplanetary space activity in the mid-21st century, Mars and Jupiter. From this orbit, a three-year transfer to Jupiter can be accomplished 100% of the time with a one-way ΔV of 40 km/s. Thus, if the goals that were laid down at the beginning of this chapter are to be met, the propulsion system would have to provide at least this much ΔV capability. Also, Venus, Earth and Mars can be reached under ΔV's of 40 km/s in three years or less, considering the points that were brought up in the discussion of the inner planets previously. Saturn can be reached

nearly 100% of the time at 40 km/s using flight times of three years, which is an added bonus. To reach Uranus with a ΔV of 40 km/s, flight times of four and five years must be used, and, for five years, the transfer is possible only 73% of the time. This was felt to be acceptable, because Uranus fell into the category of planets deemed marginally important for exploration in the mid-21st century. Mercury can be reached 100% of the time if a trip to that planet is required, but Neptune and Pluto cannot be reached at all by transfers with flight times up to five years and a ΔV limit of 40 km/s. Thus, trips to these outer planets would require special considerations, such as somehow refueling part of the way there or increasing the propulsion capabilities. A nominal orbit of 4 AU's also puts the Emerald City within reasonable distance of Earth for communications and emergencies, which is why a larger radius orbit was not selected (the benefits gained by moving the orbit out were generally negligible).

An interesting point that should be brought up when discussing the necessary ΔV that the propulsion system must provide is the return to the nominal orbit. Early in this study, it was generally assumed that accounting for the return trip would roughly double the maximum ΔV that would be necessary. However, upon further investigation, some doubts were raised as to the validity of this assumption. When using the MULIMP code, the orbit of the Emerald City is entered by using the six classical orbital elements; the eccentricity, the semi-major axis, the inclination, the longitude of the ascending node, the argument of the periapsis, and the time

of periapsis passage. For the circular orbits that were examined, there is no periapsis, and the last element is replaced by some value that locates the Emerald City in the orbit at a given time. When going from the nominal orbit to a target planet, the Emerald City will leave from a particular point on the nominal orbit and arrive at the target planet, and the ΔV can be calculated by MULIMP. However, for the return trip, there is nothing that says the Emerald City has to return to a particular point on the nominal orbit. Instead, it is free to return to any point along the nominal orbit. Unfortunately, when the orbital elements are used to define the nominal orbit, a "ghost point" is created that moves around the defined nominal orbit, as if it were a planet in that orbit. If MULIMP were used to figure out a ΔV required for the return trip, it would calculate it as if it had to return to the "ghost point's" location instead of anywhere on that orbit. Because of this freedom, the Emerald City could return to the nominal orbit at the point that would require the least ΔV for the transfer, and the ΔV's would probably not be as high as originally expected. It was not known if or how MULIMP could be used to calculate the ΔV for a situation like this, so an total ΔV of 50 km/s was assumed as the necessary upper limit to give to the propulsion system. For the inner planets, this figure should be quite adequate due to the Emerald City's ability to make optimal transfers as was discussed above. For the outer planets, it was felt that this figure should result in 100% ability to reach Jupiter and Saturn in three years and above 50% to reach Uranus in

five years (or less, in some cases). Of course, Neptune and Pluto still are unreachable without some special considerations.

2.2 ΔV MINIMIZATION

2.2.1 Introduction

The field of optimization is a powerful tool in the engineering design process. The basic optimization problem is one of determining the minimum or maximum a function, called the objective function, can obtain, subject to certain constraints on its variables. This problem is represented by

$$\min_{x \in R^n} f(x) \tag{2.2}$$

subject to the constraints

$$g_j(x) = 0 \text{ for } j = 1,\ldots.,m_e$$

$$g_j(x) > 0 \text{ for } j = m_e+1,\ldots.,m$$

$$x_l < x < x_u.$$

The function f(x) is the objective function and x is the design variable vector of length n, i.e. n design variables. There are m constraints, the first type being called equality constraints while the second type is called inequality constraints. The third type of constraint is on the design variables themselves and specifies the range of values that these variables can take. There has been much theory developed on techniques to solve optimization problems in the above form. The particular method selected depends upon the nature of the problem; the type of constraints, whether the problem is linear or non-linear, whether the gradients of the functions are to be evaluated analytically or by finite-difference methods, etc..

All of these factors will need to be considered when selecting the computer code to be used to solve the optimization problem.

With the above discussion in mind, an optimization problem studied this year will be discussed, namely the problem of ΔV minimization.

2.2.2 Background

One of the major problems that has surfaced in the first two years of development of Project WISH deals with the transfers from a nominal orbit to the planets in the solar system. It has been noted that the ΔV needed for these transfers have a great impact on the propulsion system requirements. As the values for the ΔV increase, the demands on the propulsion system become considerable. Besides straining the propulsion system, this has a cascade effect on the rest of the systems of the Emerald City; greater propulsion system requirements (thrusts, powered-flight times...) lead to more difficult problems in shielding, structural dynamics, propellant storage, etc... Thus, it is in the best interest of the Emerald City to keep these ΔV's as low as possible. For this reason, it was felt that an optimization of ΔV was justified in order to keep propulsion system requirements as reasonable as possible. It may seem that this would be redundant with the work that was presented earlier in this chapter. However, the analysis presented there used a computer program that assumed an instantaneous change in velocity, with no regard for the powered-flight times, thrust required, or the thrust angles that could be used. By performing

this optimization, it is hoped that these factors can be taken into account, as well as determining the orientation between the Emerald City and the target orbit, typically near the destination planet, that will yield the lowest ΔV.

2.2.3 Assumptions

In order to carry out the optimization, certain assumptions regarding the nature of the orbits and transfers had to be made. The first assumption is that planets are all in circular orbits and are all in the ecliptic plane, i.e. the same plane as the Earth. As was shown in Table 2.1, this is a reasonable assumption for all of the planets except Mercury and Pluto, which are of the least interest in Project WISH. The second assumption made was that there is no loss of propellant during the free-flight phase of the transfer. Of course, there will be some boil-off of the liquid hydrogen due to heat leakage through the storage tanks, but for simplicity, it was ignored for this analysis. The validity of this assumption is questionable, but it was reasoned that the results would still be applicable for this preliminary study. By making this assumption, the acceleration at the end of the first powered-flight phase was set equal to the acceleration at the beginning of the second powered-flight phase.

The third, and most pivotal, assumption is that the trajectory during the powered-flight times is unaffected by the pull of the sun's gravity (see Figure 2.3). During the powered-flight times, the Emerald City is assumed to follow a projectile-like course,

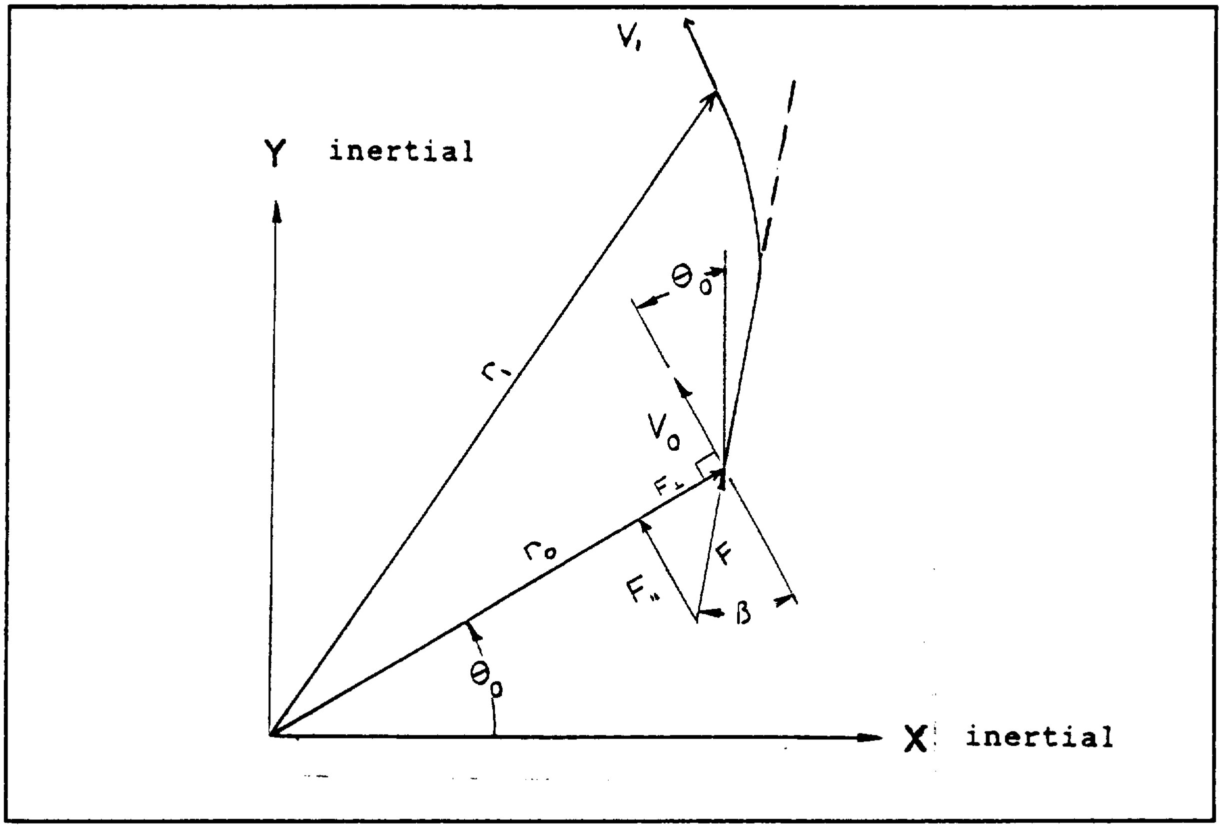

Figure 2.3 - Assumption for Powered-Flight Phases

determined by the X- and Y-components of the initial velocity and acceleration (thrust) vectors. This assumption is based on the value of the ratio of accelerations due to thrust and gravity during the powered-flight phases, given by

$$R(t) = \frac{a_{th}}{a_g} = \frac{F_{th} r(t)^2}{\mu m(t)} \qquad (2.3)$$

$$m(t) = m_o - \dot{m}t \quad \mu = 1.327 \times 10^{11}\ \frac{km^3}{s^2}.$$

It can be seen that R is dependent upon the thrust, F_{th}, the radius, r(t), and ship mass, m(t). For representative values for the Emerald City, R will be a very large number, and thus it was felt that this assumption was valid for this preliminary investigation.

2.2.4 Geometry

The geometry that was used for the ΔV optimization is shown in Figure 2.4. The technique used to model the interplanetary transfer was to break down the transfer into three arcs. The first and last arcs represents the initial and terminal powered-flight phases, respectively, while the middle arc represents the free-flight phase. In the powered-flight portions, the thrust level, powered-flight times, and thrust angles govern the trajectory, while in the free-flight portion, the classical relationships governing orbiting bodies are used (more on these relationships can be found in Appendix A). The variables shown in Figure 2.4 are defined in Table 2.13.

Table 2.13 - Definition of Variables Used in ΔV Minimization Problem

r_0	nominal orbit radius
r_1	radius at end of first powered flight, beginning of free-flight
r_2	radius at end of free-flight, beginning of second powered flight
r_f	target orbit radius
θ_0	angle between inertial reference frame X-axis and r_0 vector
θ_1	angle between inertial reference frame X-axis and r_1 vector
θ_2	angle between inertial reference frame X-axis and r_2 vector
θ_f	angle between inertial reference frame X-axis and r_f vector
θ_i	angle between inertial reference frame X-axis and target planet at launch
γ_0	angle between inertial reference frame X-axis and eccentricity vector (line of apsides)
γ_1	true anomaly of free-flight orbit at point 1
γ_2	true anomaly of free-flight orbit at point 2

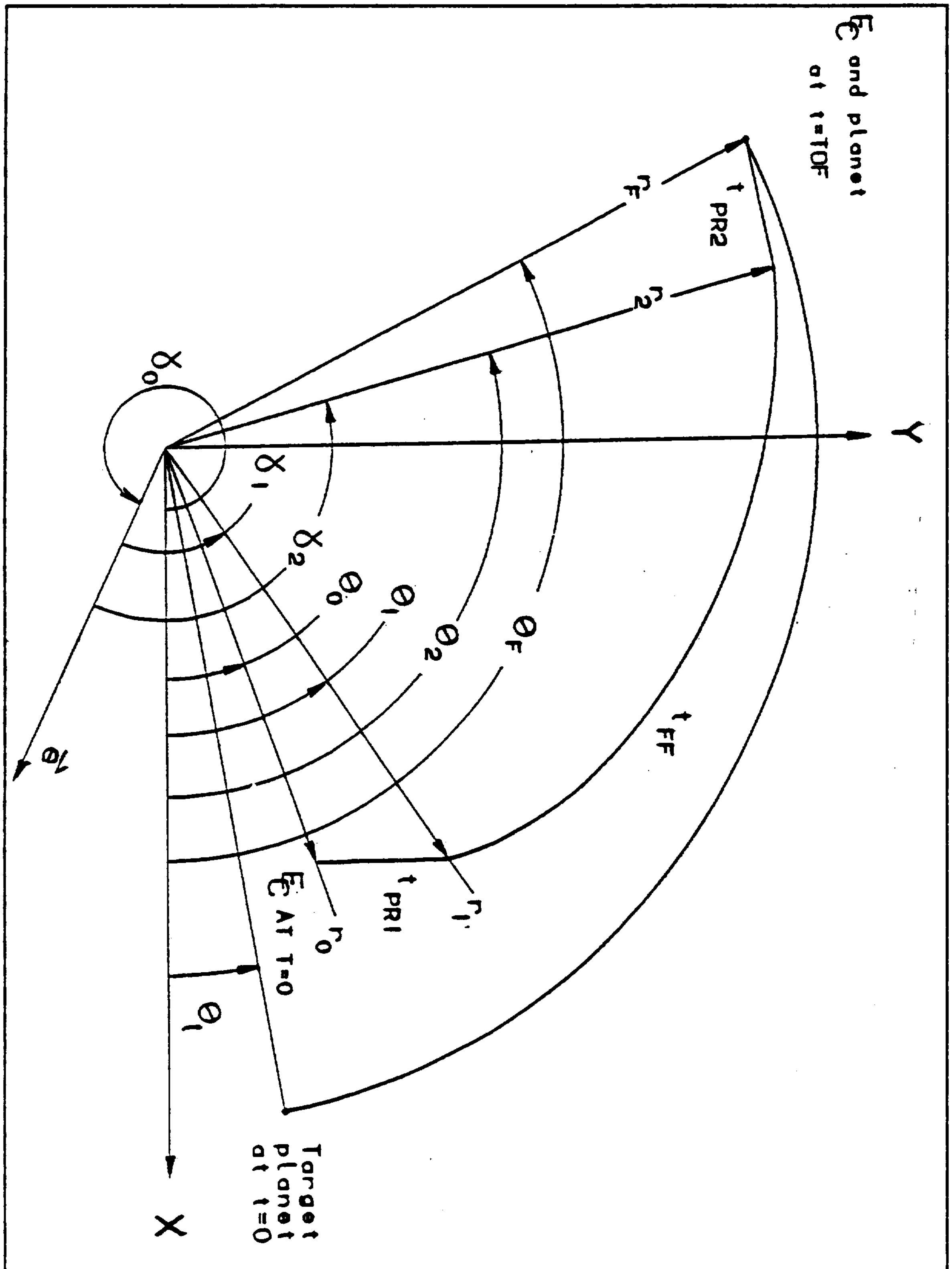

Figure 2.4 - Geometry for Optimization Problem

2.2.5 Methodology

In an optimization problem, there is a objective function that represents the variable to be optimized and constraint functions that represent the restrictions on the problem. For the ΔV minimization problem, the general objective function is

$$\Delta V_T = \Delta V_1 + \Delta V_2 = (\vec{V}_1 - \vec{V}_o) + (\vec{V}_f - \vec{V}_2) \tag{2.4}$$

where V_1, V_0, V_f, and V_2 are the velocities at points 1, 0, f, and 2, respectively (see Figure 2.4). The general constraints for this problem are given by constraints on the total time-of-flight, the acceleration levels, the powered-flight time lengths, the rendezvous requirements, and physical variable constraints. The acceleration and powered-flight-time constraints arise from the limitations of the propulsion system, while the total time-of-flight constraint governs the allowable time to make the transfer. The rendezvous constraint requires that the position and velocity of the Emerald City at the end of the transfer matches the position and velocity of the target orbit near the destination planet at that time, and the final ccnstraint is to ensure that there are no physically meaningless solutions found, such as negative times or negative magnitudes. Thus the problem is to find the equations that relate v_0, v_1, v_2, and v_f to the design variables chosen and determine equations for the constraints listed above.

The first step that was taken was to define a state vector for point i, x_i, such that

$$\vec{X}_i = \{\vec{r}_i \quad \vec{v}_i\}. \tag{2.5}$$

The values at the end of each powered-flight phase can be related to the values at the beginning of each powered-flight phase by

$$\begin{aligned} \vec{X}_1 &= \vec{X}_o + \Delta\vec{X}_1 \\ \vec{X}_f &= \vec{X}_2 + \Delta\vec{X}_2 \end{aligned} \tag{2.6}$$

where

$$\Delta\vec{X}_j = \{\Delta\vec{r}_j \quad \Delta\vec{v}_j\}. \tag{2.7}$$

By examining the equations governing the trajectory of the Emerald City during the powered and free-flight phases, and by contemplating the nature of the problem, eight design variables were identified. These variables are the two powered-flight times, t_{pr1} and t_{pr2}, the two thrust angles, β_1 and β_2, the acceleration level, a_0, the true anomaly at the end of free-flight, γ_2, and the initial orientations of the Emerald City and the target planet, θ_0 and θ_i, respectively. The functional dependencies between these variables and the state vectors were found to be

$$\begin{aligned} \vec{X}_1 &= \vec{X}_o(\theta_o) + \Delta\vec{X}_1\ (\theta_o, a_o, t_{pr1}) \\ \vec{X}_f &= \vec{X}_2(\gamma_2)|_{\vec{X}_1} + \Delta\vec{X}_2(\gamma_2, a_o, t_{pr1}, t_{pr2}) \end{aligned} \tag{2.8}$$

where the vertical bar signifies dependance on a previous state. After applying equations for ΔV_1 and ΔV_2 as found in Appendix A, the objective function was written as

$$\Delta V_T = -I_{sp}g[\ln(1 - \frac{a_o t_{pr1}}{I_{sp}g}) + \ln(1 - \frac{a_o t_{pr2}}{I_{sp}g - a_o t_{pr1}})], \tag{2.9}$$

where I_{sp} is the specific impulse of the propulsion system (see Chapter 3), and g is the gravity constant on the surface of the Earth, 9.81 m/s^2. The constraint functions may then be written as

1. $t_{ff} + t_{pr1} + t_{pr2} < TOF_{max}$
2. $a_0 < a_{0,max}$
3. $t_{pr1} + t_{pr2} < t_{pr,max}$
4. $x_{f,EC} = x_{f,TO}$.
5. a_0, t_{pr1}, t_{pr2}, $t_{ff} > 0$.

It turns out that the objective function is dependant only on three of the eight design variables, namely a_0, t_{pr1}, and t_{pr2}. The other five design variables enter into the problem through the first and fourth constraints above. These constraints on the total time-of-flight and the rendezvous requirement incorporate the rest of the design variables through the equations governing the powered and free-flight phases of the trajectory. These equations can be found in detail in Appendix A.

Once the objective function and constraint functions are determined, an optimization code can be utilized to solve this problem. A potential optimization subroutine was identified, but due to time and personnel constraints, the problem was not programmed. This subroutine solves a general, non-linear optimization problem using a successive quadratic programming algorithm and finite difference gradients. It is hoped that this optimization problem can be programmed during next year's design effort and some meaningful results obtained.

2.3 CONCLUSIONS

This year, a detailed study was performed on the orbital mechanics of Project WISH. Based on this study, a nominal orbit of 4 AU's was selected, which places the Emerald City roughly 600,000,000 kilometers from the sun, between the orbits of Mars and Jupiter. From this orbit, it was determined that the planets Mercury, Venus, Earth, Mars, Jupiter, and Saturn can be reached 100% of the time in flight times of three years or less with a ΔV limit of 40 km/s one-way. Uranus can be reached 73% of the time with this ΔV in flight times of five years, which was felt to be adequate for this project. Neptune and Pluto cannot be reached at all at this ΔV with flight times of up to five years, but it was believed that there would be little need for a ship such as the Emerald City to make trips to these planets during the time frame considered. The ΔV minimization problem was also examined and equations were generated that yield the trajectory of the Emerald City during both the powered- and free-flight phases. The objective and constraint functions were determined, but due to a lack of time, the problem was not programmed into an optimization code. It is hoped that in the future, this problem can be solved and meaningful results can be obtained. Also, more work should be done on determining the ΔV's required to return to the nominal orbit from a planet. With additional study in this area, a more accurate round-trip figure for the ΔV's can hopefully be obtained. As of the end of this study, a total round-trip ΔV of 50 km/s was used as the upper ceiling for the Emerald City.

CHAPTER 3

PROPULSION

3.0 INTRODUCTION

The propulsion system of the Emerald City plays an important role in the success of Project WISH. For this reason, it was one of the most rigorously studied subsystems this year. At the conclusion of the Phase I design conducted last year, an antimatter engine was selected as the main propulsion system of the Emerald City[2]. However, it was decided during Phase II that antimatter was too theoretical for the time frame of Project WISH, and that the propulsion system chosen should be more readily procurable by the mid-21st century. With the hope of bringing the choice of the propulsion system into the Emerald City's time frame, the propulsion team looked at several areas of interest in spacecraft propulsion. First, a general level study was conducted in order to determine some of the important parameters associated with spacecraft propulsion and what impact they had on the choice of a particular system. Once this was accomplished, a propulsion system was chosen and a specific propulsion study was conducted in order to determine the significant characteristics of this system. From this study, it was possible to calculate other important parameters that were dependent upon the choice of a particular propulsion system. Finally, this propulsion system was used in the design of two representative missions, which may be found in Chapter 6.

3.1 GENERAL PROPULSION STUDY

3.1.1 Theoretical Background

The purpose of this general level study is to determine which parameters play a major role in the choice of a propulsion system, and what the orders of magnitude of these parameters need to be for an application such as Project WISH. The study is independent of the type of engine that will be used for Project WISH; only the physical nature of spacecraft propulsion enters into this investigation. The first step in the analysis was to determine the effects that the mission requirements have on propulsion parameters. Since the Emerald City is expected to be a large spacecraft, the propulsion system will be required to produce high impulse levels, J, at high specific impulses, I_{sp}. High specific impulse, which is the impulse produced per unit weight of propellant expelled, is desirable because it corresponds to less propellant required to accomplish a mission. The I_{sp} plays an important role in the equation for the propellant mass ratio, given by

$$\frac{m_p}{m_o} = 1 - e^{-\frac{\Delta V}{I_{sp} g}} \tag{3.1}$$

which is plotted in Figure 3.1. This figure demonstrates how the propellant mass ratio, which is the fraction of the total initial ship mass consisting of propellant, decreases as the I_{sp} is increased or the ΔV is decreased. Thus, to keep the propellant mass required for a given mission down, a propulsion system capable of generating a high I_{sp} should be utilized.

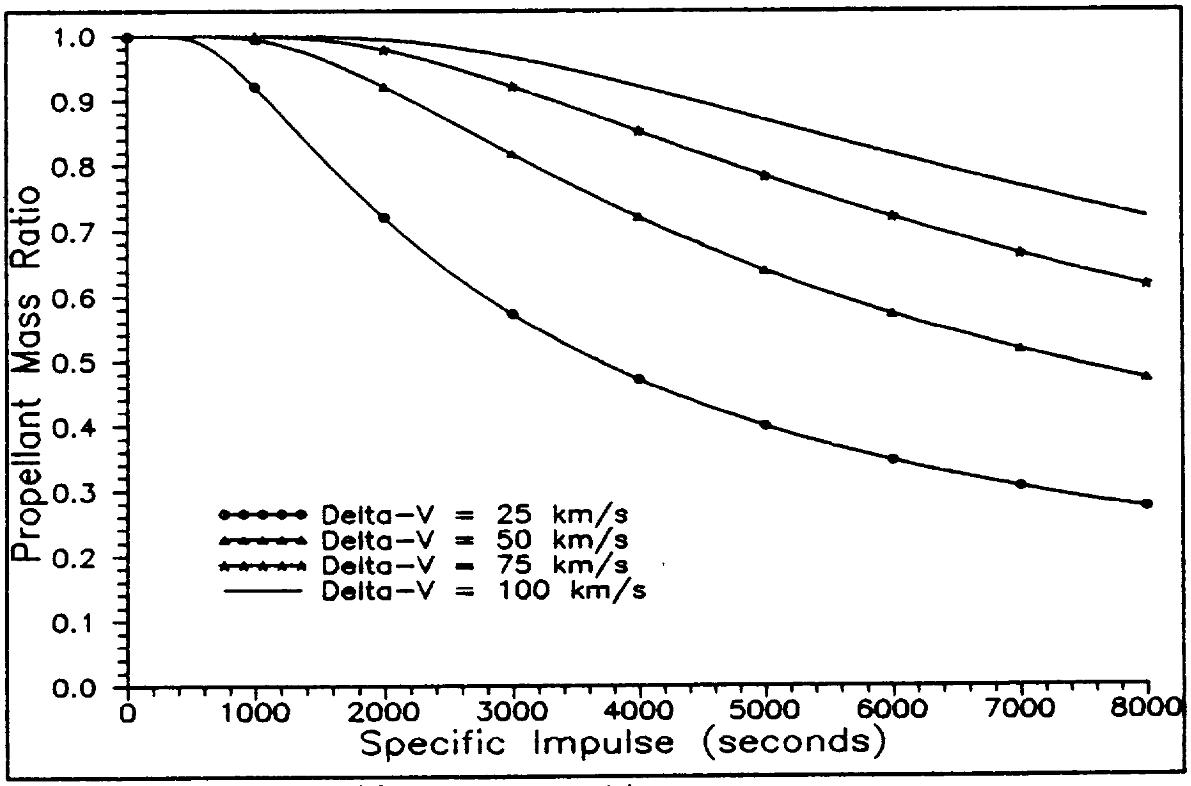

Figure 3.1 - Propellant Mass Ratio vs. I_{sp} With ΔV as a Parameter

However, for the Emerald City, a high-I_{sp} engine alone is not enough. An engine capable of producing high impulse levels as well as the high I_{sp}'s is needed. The impulse required for a mission is a function of the dry mass of the ship (no propellant included) and the ΔV and is given by

$$J = m_{dry}\, I_{sp}\, g\, \left(e^{\frac{\Delta V}{I_{sp}\, g}} - 1\right) \tag{3.2}$$

which is plotted in Figure 3.2. From this figure, it can be seen that the impulse required for an Emerald City mission will be very large, on the order of 10^{13} to 10^{14} newtons. This is due to two factors; the large dry mass of the Emerald City, and the high ΔV levels that will be required for its missions (see Chapter 2). The

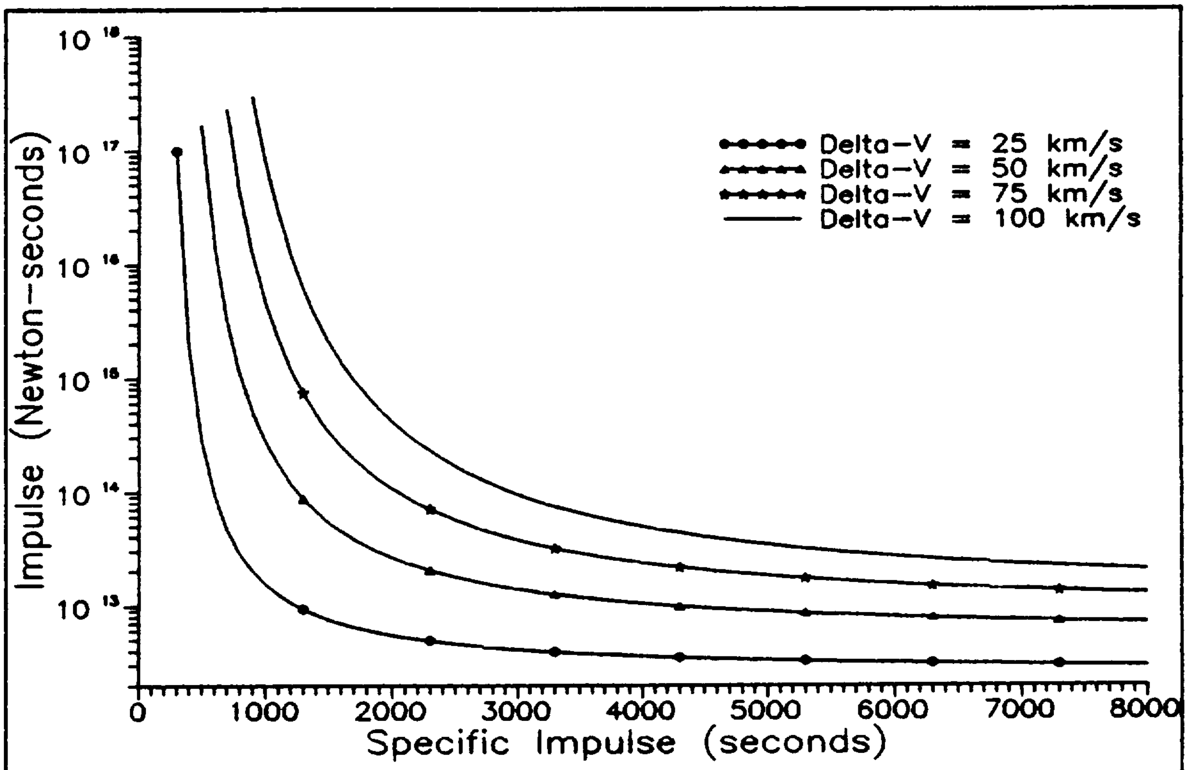

Figure 3.2 - Impulse vs. Specific Impulse with ΔV as a Parameter and $m_{dry} = 10^8$ kg

effect that the I_{sp} has on the impulse required can also be seen from Figure 3.2. As the I_{sp} goes down, the impulse required goes up, eventually approaching infinity as I_{sp} approaches zero. This is because of the extra propellant mass that must be carried at the lower I_{sp}'s, which makes the Emerald City heavier and more difficult to move. Since impulse is the product of a force and a time, high impulses would necessitate high thrusts, F, and high powered-flight times, t_{pr}. Neither of these are desirable from an engine maintenance/lifetime viewpoint. It may also be seen from Figure 3.2 that, if the Emerald City were to burn for its entire flight time (~ 3 years or ~ 10^8 seconds), the thrust that would be required would still be on the order of 10^5 newtons.

Once it has been determined that high I_{sp}'s and high thrusts are needed, the next step was to search for a propulsion system that could satisfy these requirements. A general level study was conducted in an attempt to find such a system from choices that included chemical, electrical, nuclear, and antimatter. However, antimatter had already been ruled out for being too theoretical in nature, even for an advanced project such as Project WISH. Thus, the remaining systems were analyzed in terms of the specific impulse and thrust levels that each system could provided.

3.1.2 General Propulsion Systems

Chemical propulsion was the first type of propulsion analyzed and was promptly rejected. Although the thrust values were adequate, the specific impulses of these systems fell far short of the requirements that the Emerald City would place on them. Electrical propulsion was the next system that was studied. These systems produced high enough I_{sp}'s, but were severely limited in the amount of thrust that could be produced. Nuclear thermal propulsion (NTP) was the final type of system examined. Estimates of the nuclear propulsion systems of the 21st century gave the best combination of high thrust and high I_{sp} that would be required for Project WISH. Thus, NTP was examined in more detail as a primary propulsion system for the Emerald City.

One of the main points considered when examining nuclear systems was the question of fission or fusion. The propulsion group decided that fusion, like antimatter, would be too

theoretical for the Emerald City. Another drawback of using fusion propulsion is the mass penalty that would accompany the system. Compared to the fission systems, the fusion systems are estimated to be substantially more massive. Of course, a breakthrough in fusion research could lead to the development of an operational fusion system suitable for the Emerald City, but for now, fusion was rejected as a main propulsion candidate. Fission systems have the advantage of being more advanced and currently less massive than their fusion counterparts. Unfortunately, associated with fission is a rather troublesome radiation problem, which leads to an increase in the shielding mass required for human safety. Also, the more developed types of fission systems have relatively low I_{sp}'s for this project, and the more advanced engines with the higher I_{sp}'s are still conceptual in nature. With all of these considerations in mind, it was decided that a fission nuclear propulsion system would be used for the Phase II design of the Emerald City.

Within the fission category of NTP, there are two major subgroups; solid-core reactors and gas-core reactors. As its name suggests, a solid-core NTP system utilizes a fissioning solid-core of uranium to generate thermal energy. This energy is transferred to a lightweight propellant, usually hydrogen, which is then expelled through a convergent-divergent nozzle. However, the core is limited in temperature by material considerations, and thus the achievable I_{sp} is similarly limited. Current estimates and some testing during the NERVA (Nuclear Energy Rocket Vehicle Appli-

cation) place the solid-core I_{sp} in the range of 800-1100 seconds[5]. The gas-core NTP system attempts to overcome this issue of limited temperature by allowing the core temperature to increase beyond the boiling point of uranium, thus resulting in a gaseous core. This increase in temperature enables more thermal energy to be transmitted to the propellant, which results in a higher possible I_{sp}. Current theoretical studies give a possible I_{sp} for this conceptual engine on the order of 2000-8000 seconds[6]. It was felt that the I_{sp} of the solid-core rockets were simply too low for the needs of Project WISH (see Figures 3.1 and 3.2) and thus the gas-core NTP system, with its higher estimated attainable thrusts and I_{sp}'s was chosen for further study as the main propulsion system of the Emerald City.

There were two major types of gas-core nuclear rockets (GCNR) studied, the light bulb (closed-cycle, LBGCNR) and space radiator (open-cycle, SRGCNR). The closed-cycle gas-core engine has its gaseous core encased in a transparent material that allows radiation from the fissioning core to heat the propellant while keeping the propellant and core physically apart. The open-cycle gas-core engine has the propellant and core in direct contact, which allows for better heat transfer, but also leads to a loss of uranium from the core. In determining which of these propulsion systems was best for the Emerald City, three main engine parameters were investigated, the I_{sp}, the thrust per engine, F_i, and the mass per engine, m_{wi}. Table 3.1 shows some comparison figures of these three parameters for the two systems considered[6]. This table shows that

the closed-cycle engine has a significantly lower specific impulse than the open-cycle engine. However, the maximum expected thrust value for the closed-cycle system is higher than that of the open-cycle engine. It will be shown later in this chapter in Figure 3.5 that the advantage of producing higher thrust decreases as the thrust level increases. It was felt that the closed-cycle gas-core engines still had I_{sp} values that were too low for use by the Emerald City, and thus, the open-cycle engine (hereafter referred to as SRGCNR) was chosen as the propulsion system for the Emerald City. The values of thrust and I_{sp} that were used as representative figures for the SRGCNR in the remainder of this report are[7]

Table 3.1 - Comparison of I_{sp}, F, and m_e for LBGCNR and SRGCNR

LBGCNR DATA

Thrust (N)	m_{vi} (kg)	I_{sp} (sec)
133,370	14,050	1780
1,334,200	34,475	2355
4,002,800	385,500	2635

SRGCNR DATA

Thrust (N)	m_{vi} (kg)	I_{sp} (sec)
22,240	36,280	2400
177,900	101,440	5500
444,750	213,350	6000

$$I_{sp} = 5000 \text{ seconds}$$

$$F_i = 4.4 \times 10^5 \text{ N.}$$

These values fell within the range of thrusts and I_{sp}'s that were discussed in Reference 7, and were felt to be reasonably obtainable if the SRCGNR is ever developed. Once the propulsion system and important parameters were determined, it was then possible to conduct a detailed study of the SRCGNR system of the Emerald City.

3.2 SPACE RADIATED GAS-CORE NUCLEAR ROCKET STUDY

With the specific propulsion system determined and values for I_{sp} and F_i known, it was then possible to perform a more detailed study of this system. Because the SRCGNR is so conceptual in nature, there are many unanswered questions about its form and function. Figure 3.3 is a schematic of the GCNR that was conceptually developed at NASA Lewis Research Center by Mr. Robert Ragsdale[7]. The main mechanical components of this engine are the pressure shell, the moderator, the turbopump, the nozzle, and the

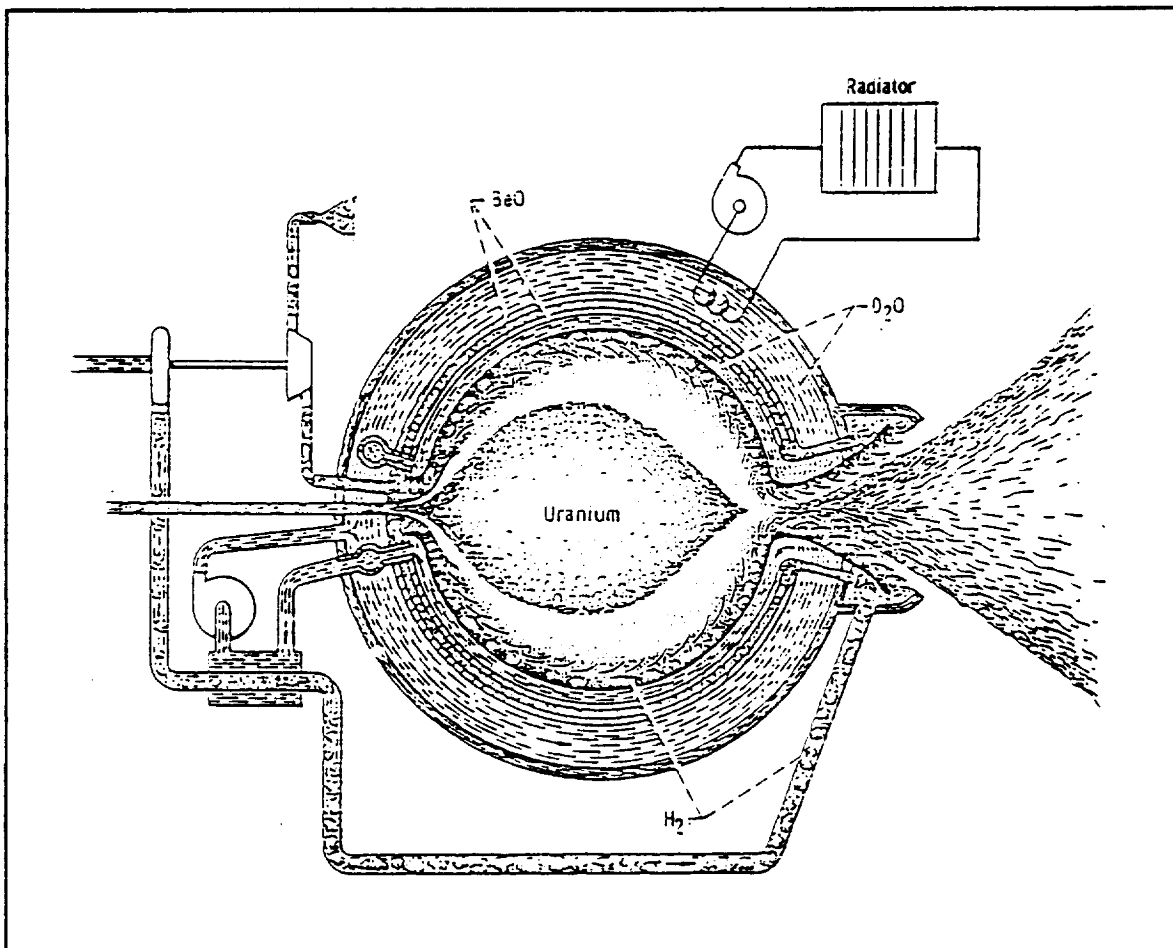

Figure 3.3 - Conceptual Sketch of the Gas-Core Nuclear Rocket

radiator, which work together in the following manner. First, liquid hydrogen is pumped through the moderator by the turbopump to aid in heat removal there. Then, the hydrogen is pumped into the cavity where the uranium isotope, U^{233}, is undergoing the fission process. The hydrogen absorbs heat from the fissioning uranium and is expelled through the nozzle to produce thrust. The moderator is used to reflect and reduce the energy of (thermalize) the neutrons which aids in sustaining the fission process. The pressure shell encloses the core, cavity, and moderator, while the radiator is used to remove any excess heat that builds up in the engine which cannot be removed by the hydrogen alone.

Once a working knowledge of the engine had been acquired, it was then possible to compute various important engine characteristics. These computations were done using a computer program written during this study, which can be found listed in Appendix B. This program was used to generate the important parameters of the SRCGNR from general equations and equations given by Mr. Ragsdale for estimating the mass of an engine[7]. The results obtained from this program may be found tabulated in Table 3.2. The program has four inputs that need to be determined before the other parameters may be calculated. These inputs are I_{sp}, F_i, cavity diameter, D_c, and the ratio of the volume of the gaseous uranium core to the volume of the cavity, V_u/V_c. For the data in Table 3.2, these values were selected to be I_{sp} = 5000 seconds, F_i = 4.4 x 10^5 newtons, D_c = 3.3 meters, and V_u/V_c = 0.23. These figures were considered to be reasonable estimates of the operating conditions

Table 3.2 - Engine Parameters of a SRGCNR, $F_i = 4.4 \times 10^5$ N, $I_{sp} = 5000$ s, $D_c = 3.3$ m, and $V_u/V_c = 0.23$

SPECIFIED VALUES:

I_{sp}=5000.00 s   Thrust=440000.00 N V_u/V_c=.23 D_c=3.30 m

CALCULATED ENGINE PARAMETERS:

ENGINE CAVITY CONDITIONS:

$T(H_2)$=25165.49 K	$T(U^{233})$=76183.45 K
Cavity Pressure=981.25 atm	Critical Mass U^{233}=40.12 Kg
Cavity Volume=18.82 m^3	Uranium Volume=4.33 m^3
Uranium Diameter=2.02 m	Uranium Density=9.27 Kg/m^3
Hydrogen Molecular Mass=0.759	Gamma=1.261

MASS FLOW RATES:

H_2-To-U^{233} Ratio=393.89	Propellant=8.97 kg/s
H_2=8.951 kg/s	U^{233}=0.023 kg/s

ENGINE DIMENSIONS:

Pressure Shell Thickness=0.087 m	
Pressure Shell Mass=52597 kg	
Moderator Thickness=0.76 m	Moderator Mass=46632 kg
Turbo Pump Mass=773 kg	Exhaust Nozzle Mass=233 kg
Radiator Mass=138463 kg	Total Engine Mass=238682 kg

VARIOUS ENGINE PARAMETERS:

Reactor Power=16185.08 MW	Radiated Power=971.10 MW
Specific Mass=0.02224 kg/KW	Jet Power=10787.32 MW
Thrust to Weight Ratio=0.188	Engine Efficiency=0.6665

THROAT CONDITIONS:

Temperature=22258.87 K	Pressure=524.48 atm
Hydrogen Molecular Mass=0.843	Gamma=1.246
Area=0.0022 m^2	Velocity= 16542.23 m/s

EXIT CONDITIONS:

Temperature=4120.51 K	Pressure=0.115 atm
Hydrogen Molecular Mass=2.004	Gamma=1.226
Area=0.6489 m^2	Velocity= 49033.25 m/s
Exit-To-Throat Area Ratio=300	Mach Number= 6.50

of the SRGCNR engine[7]. With these inputs, the program was able to calculate the conditions in the cavity, in the throat, and at the exit of the engine. The formulas that were used to calculate these conditions are included with the program in Appendix B. Some of the more interesting results noted from Table 3.2 are a throat temperature of 22,260 K, a jet power of 10.8 gigawatts, a reactor power of 16.1 gigawatts, a thrust-to-weight ratio of 0.19, and an exit velocity of 49 km/s. This value of exit velocity is over ten times what a standard chemical rocket can attain today. The overall mass of the engine was found to be 238.7 metric tonnes.

Once the SRCGNR characteristics and values were obtained, it was then possible to proceed with other calculations that were related to the propulsion system. For example, the total number of engines, n, that would be required for any given mission had to be determined. This formula, given by

$$n = \frac{m_{dry}\, I_{sp}\, g}{F_i\, t_{pr}} \left(e^{\frac{\Delta V}{I_{sp} g}} - 1\right) \tag{3.3}$$

was found by replacing J in Equation 3.2 with $nF_i t_{pr}$. Once the number of engines was determined, it was possible to calculate the total engine mass from

$$m_w = n\, m_{wi}. \tag{3.4}$$

Equation 3.3 is plotted in Figure 3.4, which shows the relationship between the number of engines required and the mission ΔV for a range of different powered-flight times. This figure was generated using an I_{sp} of 5000 seconds, an F_i of 4.4×10^5 newtons,

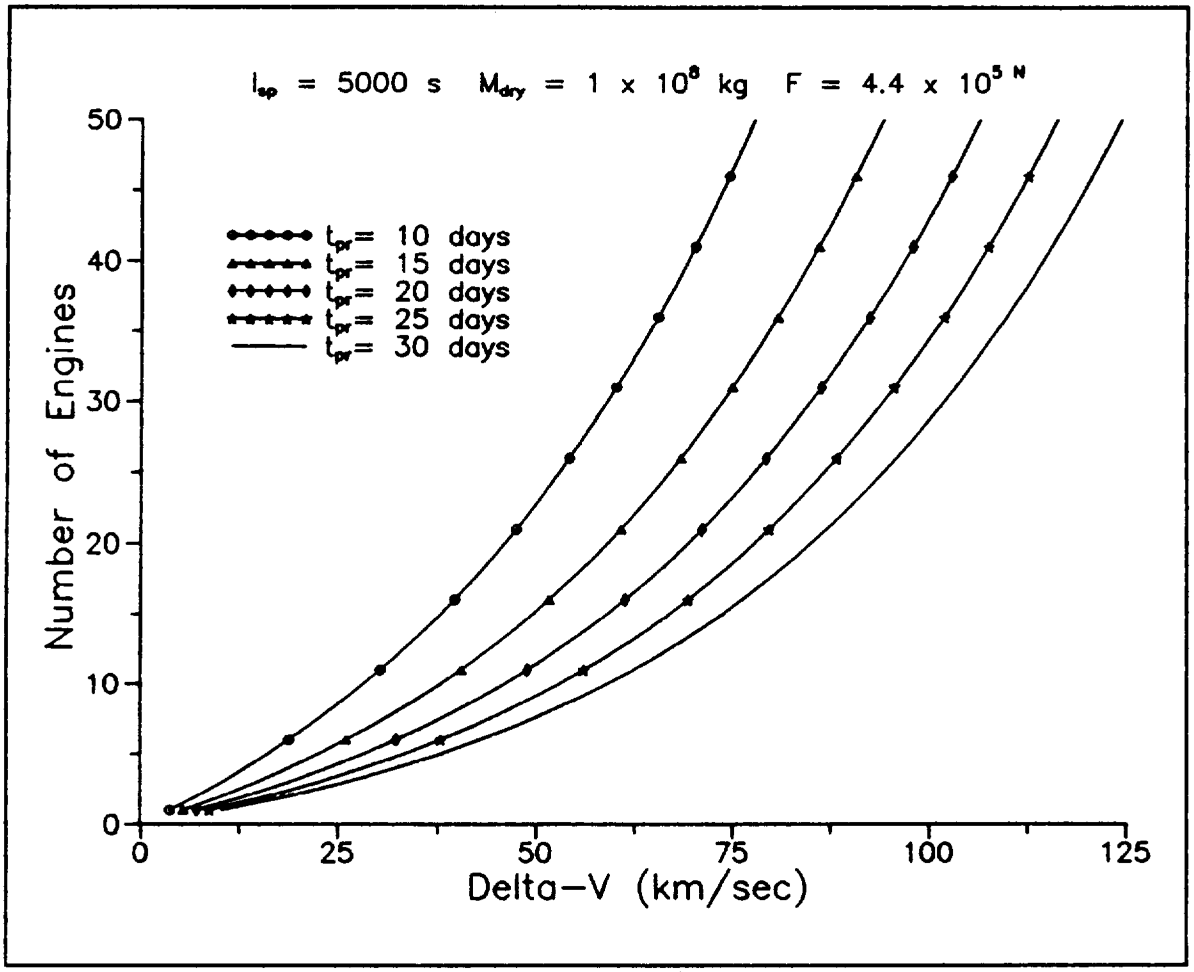

Figure 3.4 - Number of Engines vs. ΔV with Powered-Flight Time a Parameter

and an assumed dry mass of 1×10^8 kg. By using Equation 3.3, the number of engines for any selected mission parameters (ΔV, t_{pr}, m_{dry}) may be determined. Once the total engine mass has been found, it was then possible to calculate the payload mass in the form of the payload mass ratio, given by

$$\frac{m_l}{m_o} = e^{-\frac{\Delta V}{I_{sp} g}} - \frac{I_{sp} g}{I_{spe}} \left(1 - e^{-\frac{\Delta V}{I_{sp} g}} \right). \qquad (3.5)$$

In Equation 3.5, I_{spe} is defined as the engine specific impulse and is defined as the impulse produced per unit mass of the engine, or

$$I_{spe} = \frac{F_i \, t_{pr}}{m_{wi}} \tag{3.6}$$

Equation 3.5 was used to generate Figure 3.5, which graphically shows the relationship between the payload mass ratio and the engine specific impulse I_{spe} for different mission ΔV's. An interesting note about this plot is that the payload mass ratio

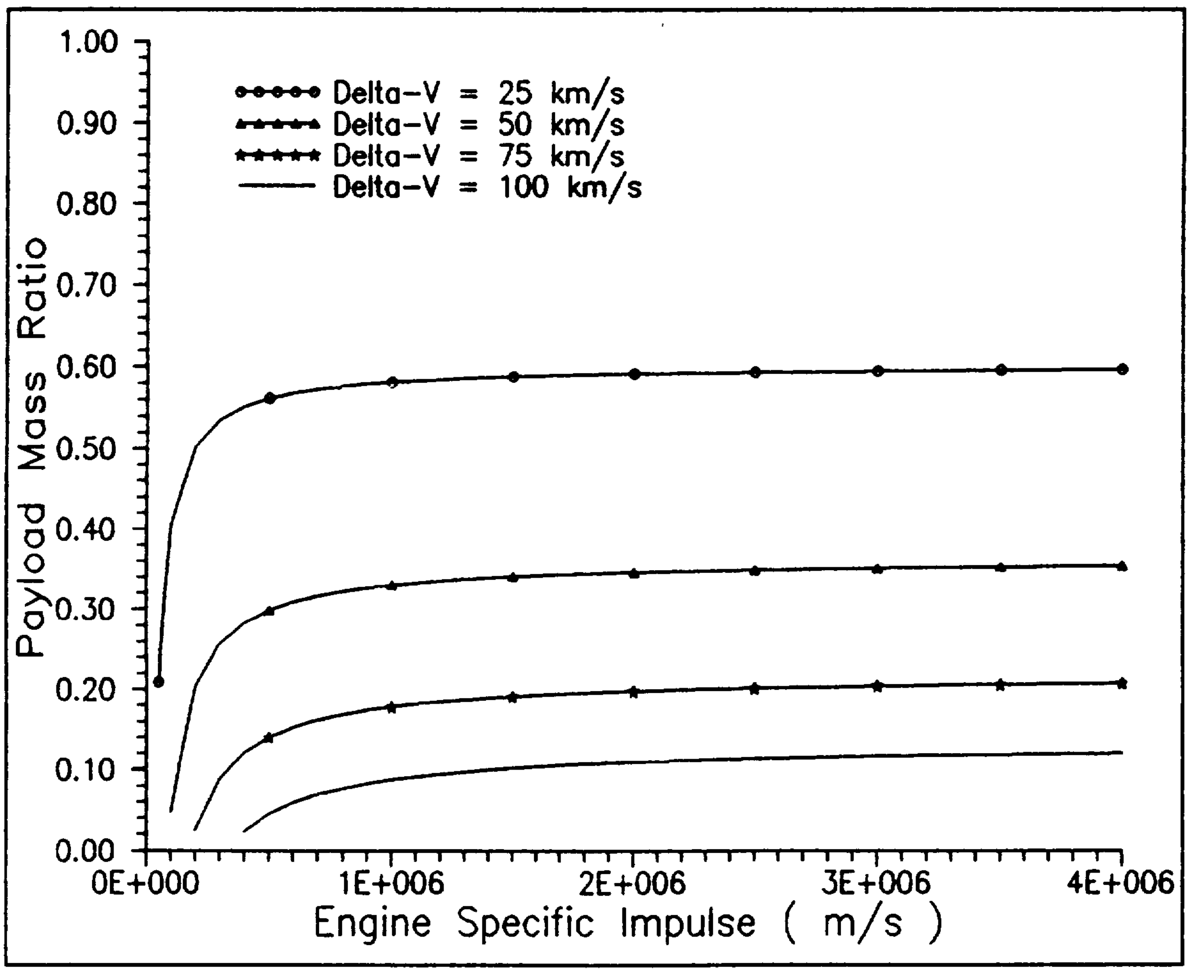

Figure 3.5 - Payload Mass Ratio vs. Engine Specific Impulse (I_{spe}) with ΔV as a Parameter.

tends to level out no matter how much the engine specific impulse is increased. Thus, an increase in the I_{spe} is only beneficial when operating in the region where the payload mass ratio substantially increases. There is little to be gained by increasing the thrust or the powered-flight time or decreasing the engine mass once the I_{spe} has reached the point where the payload mass ratio begins to level off. Also, it may be noted from Equation 3.5 that there is a value of I_{spe} below which, the engine mass exceeds the entire payload mass, and thus the payload mass ratio will be negative.

3.3 CONCLUSIONS

The end result of the analysis presented in this chapter was to find equations that would yield the total mass of the propulsion system and propellant required. By using the equations and figures given here, these masses can be found for different mission specifications. This is a crucial step in the design of the Emerald City for specific missions. Two missions were designed for the Emerald City, using the equations derived here. These missions can be found in Chapter 6.

The preliminary propulsion system that was chosen was the space radiated open-cycle gas-core nuclear rocket engine. The benefits of this system are its high I_{sp} and thrust values that can be obtained. However, this engine is still very conceptual in nature, and so it is not currently known if the idea will eventually lead to a functioning rocket engine or not. It can be seen from Table 3.2 that the temperature in the throat of the

nozzle is predicted to be around 22,000 K!. Clearly, advances will have to be made in the development of materials capable of withstanding this kind of thermal load. Another area in need of development is the confinement of the gaseous core. Currently, it is envisioned that the core will be confined by a vortex of inert gas. This area will need to be thoroughly researched if the concept of an open-cycle gas-core reactor is ever to reach fruition.

There were two detrimental side effects noted by using the open-cycle engine, both of which are related. The first of these effects was the loss of uranium out of the cavity and through the nozzle. It was estimated that approximately 2 metric tonnes of uranium will be lost per engine per day of powered-flight time. The second side effect is the environmental and human factors concern of the radiation generated by this type of engine. Large amounts of radiative material are present in the exhaust plume which will result in massive shielding requirements for the crew. Both of these problems can be lessened by going to a light-bulb gas-core engine, which is a closed-cycle type of engine. However, this engine is not foreseen to have an I_{sp} as high as that of the open cycle engine. The light bulb engine could prove useful if the scope of Project WISH were to ever be reduced, i.e. fewer people, less massive ship, smaller ΔV's... Until then, however, the open cycle gas-core engine will remain as the only type of nuclear engine capable of generating the high impulse-high I_{sp} combination needed for Project WISH in the mid-21st century.

CHAPTER 4

ATTITUDE CONTROL

4.0 INTRODUCTION

Attitude control is an important facet of Project WISH due to the fact that it will be necessary to damp out oscillations in attitude due to disturbances. Because the Emerald City will be spinning to produce artificial gravity (see Chapter 5), the system will be gyroscopic in nature. Therefore, stable equilibrium configurations had to be determined, which was done during the Phase I study conducted last year. Once these configurations were known, attitude control studies were conducted in order to find the control requirements needed to return the Emerald City to equilibrium following a disturbance. The methodologies used to perform these attitude control studies are the focus of this chapter.

4.1 ATTITUDE DYNAMICS BACKGROUND

For a body that is as large and complex as the Emerald City, there will be some structural deformations that arise due to the loads encountered. However, for the purpose of this preliminary investigation, the Emerald City was considered to be a rigid body, i.e. there are no deformations present. Although structural dynamics was not considered in this analysis, it was examined last year using a lumped-mass modelling method, and will again be considered in detail during the third year (1991-1992) of the

project. Since the goal of the attitude control study was to obtain an order of magnitude estimation of the control thruster requirements, it was felt that the rigid body assumption was adequate.

As stated earlier, the stable equilibrium configurations for the Emerald City were determined from last year's vehicle dynamics study[2]. These stable configurations were found to be dependent on the parameters b and r in the following manner:

1. $b > 1/r$ and $b > 4/r - 3$
2. $b < 1/r$ and $r > 1$
3. $b < 4/r - 3$ and $r < 1$.

The parameters b and r are given by

$$r = I_z / I_x = \text{slenderness ratio}$$

$$b = n / \Omega = \text{spin rate / orbital angular rate}$$

where I_z and I_x are the mass moments of inertia about the z- and x-axes (see schematic in Chapter 6), and n is the spin rate of the torus. Thus, it may be seen that the stability of the Emerald City depends upon three principal factors: mass moments of inertia (determined by ship configuration), spin rate (determined from artificial gravity considerations), and orbital angular velocity (determined by the nominal orbit).

4.2 CONTROL SYSTEM REQUIREMENTS

Once the stable configurations of the Emerald City are known, the control aspect of the problem may be examined. The main goal

of the attitude control study was to determine the values of thrust, root-mean-square power, and propellant mass required to damp out an initial disturbance.

4.2.1 State Feedback Control Design

The first step in achieving this goal was to determine the state-space equations that model the motion of the Emerald City. This equation was expressed in the non-dimensional state-space formulation[8]

$$\dot{\hat{x}} = [A][\hat{x}] + [B][\hat{T}] \qquad (4.1)$$

where

$$[A] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ (1-r) & 0 & 0 & (2-r) \\ 0 & (1-r) & (r-2) & 0 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (4.2)$$

$$[\hat{x}] = [\hat{\theta}_1 \quad \hat{\theta}_2 \quad \dot{\hat{\theta}}_1 \quad \dot{\hat{\theta}}_2]$$

and $[\hat{T}]$ is the non-dimensional torque provided by the control thrusters (^ denotes non-dimensionalized quantities). Equation 4.1 is used to simulate the state response of the Emerald City for given ship configurations, thrust input configurations, and initial disturbances. A linear quadratic regulator control program (MATLAB[9] software) was used to obtain the non-dimensional state

response that minimizes the control design performance index, defined by

$$CDPI = \frac{1}{2}\int_0^\infty (\hat{X}^T\hat{X} + \hat{T}^T\hat{T})\,d\tau. \tag{4.3}$$

By minimizing this performance index, the program minimizes the total control effort and the kinetic energy of the system and yields a linear state feedback control law

$$[\hat{T}] = [G][\hat{x}] \tag{4.4}$$

where G is the control feedback gain matrix minimizing the CDPI in Equation 4.3.

4.2.2 Attitude Control Power Required

For the control torques of Equation 4.4, the non-dimensional control power consumed can be found by the non-dimensional power S^*,

$$S^* = \int_0^\infty \hat{T}^T \hat{\boldsymbol{\theta}}^{-T} \hat{\boldsymbol{\theta}}^{-1}\hat{T}\,d\tau \tag{4.5}$$

which yields

$$S^* = \hat{X}_o^T P^* \hat{X}_o \tag{4.6}$$

for any initial disturbance state x_0. The power matrix P^* is the solution of the linear matrix equation

$$A_{cl}^T P^* + P^* A_{cl} + G\,\hat{\boldsymbol{\theta}}^{-T} \hat{\boldsymbol{\theta}}^{-1} G = 0 \tag{4.7}$$

where

$$A_{cl} = A + BG \tag{4.8}$$

and the non-dimensional thruster distribution matrix is defined by

$$\hat{\mathfrak{D}} = D_t^{-1}[D] \tag{4.9}$$

in which D_t is the torus diameter and the elements of [D] are the moment arms of respective input forces yielding moments about the centroidal body axes x and y of the vehicle which are parallel to the torus plane. The required quantities and matricies in Equations 4.4 through 4.9 can be evaluated by using any suitable control software such as the MATLAB Control Toolbox mentioned previously.

The various dimensional and non-dimensional quantities are related by

$$[\,\theta_1 \;\; \theta_2 \;\; \dot{\theta}_1 \;\; \dot{\theta}_2\,] = [\,\hat{\theta}_1 \;\; \hat{\theta}_2 \;\; \hat{\dot{\theta}}_1 \;\; \hat{\dot{\theta}}_2\,]\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & n & 0 \\ 0 & 0 & 0 & n \end{bmatrix} \tag{4.10}$$

$$t_{con} = \frac{\tau_c}{n} \tag{4.11}$$

$$F_i = \hat{F}_i \frac{I_z\, n^2}{D_t}. \tag{4.12}$$

$$T = \hat{T}\, I_z\, n^2. \tag{4.13}$$

The non-dimensional control thrust forces can be obtained via

$$\hat{F}_{con} = \hat{D}^{-1}\hat{f} \tag{4.14}$$

In all of the above equations, a ^ above a variable denotes non-dimensional quantities. The root-mean-square power required to damp out a disturbance can then be found from the value of S^*, computed via Equations 4.6 and 4.7. The root-mean-square control power required is then given by the formula

$$P_{rms} = \frac{v_{ex} I_s\, n^2}{2D_t}\sqrt{\frac{S^*}{\tau_c}} \tag{4.15}$$

where v_{ex} is the exit velocity of the control thrusters.

4.2.3 Attitude Control Propellant Requirement

The propellant mass required for the control thrusters can be found by the formula

$$m_{p,con} = \left(\frac{I_s\, n}{D_t\, v_{ex}}\right) \sum_{j=1}^{f} \int_0^{\tau_c} |\, \hat{F}_j \,|\, d\tau \tag{4.16}$$

where f is the total number of inputs used. The integral in Equation 4.16 is found by numerically integrating the non-dimensional thrust input versus non-dimensional time plots (see for example Figure 6.4 for specific mission designs)

4.2.4 Attitude Control Thruster Configuration

For a body with the size of the Emerald City, one would need hundreds, perhaps thousands, of control thrusters to exert attitude control torques about the body axes. The number would depend on

the thrust output capabilities of the control thrusters. Taking into account so many thrusters individually in the attitude control problem study would result in a very large D (thruster distribution) matrix. However, because the Emerald City is assumed rigid, one can instead consider only a few clusters of thrusters, where each cluster is represented as if it is a single thrust force obtained as the resultant force of hundreds of thrusters each producing identical thrust-time histories in a given cluster location. Thus, each cluster location acts as one input in the control study. After finding the total thrust-time history at a particular location through the control study, one can then apportion equally the required thrust to individual thrusters. Thus, for given thrust capabilities of thrusters, the total number of thrusters required in a particular cluster can be found subsequently. For example, in Figure 6.5, only three cluster locations are shown as if there are only three control thrust inputs, F_1, F_2, and F_3, and the attitude control requirements are computed for that particular input configuration. How many individual thrusters may exist at each cluster becomes immaterial at this point and the control study can proceed.

4.3 ATTITUDE CONTROL DESIGN METHODOLOGY

The methodology developed in this chapter can be used to study the attitude control of the Emerald City for a wide range of ship configurations, thruster configurations, and initial disturbances. Two particular scenarios were examined and the control system

requirements were determined using the methods provided above. The results for particular mission studies can be found in Chapter 6. The outline of the methodology as described above is reviewed here in step-by-step form.

1. Choose a cluster configuration, characterized by Equation 4.9, to be studied and determine the slenderness ratio, r, from the ship configuration. Select an initial mean disturbance to investigate.
2. Use MATLAB (or a similar linear quadratic control regulator program) to determine the feedback gain, G, and the state response, x, to Equation 4.1 that minimizes the control design performance measure of Equation 4.3.
3. Use MATLAB to find the control torque, T, and the resultant cluster inputs, F (via Equation 4.14), versus time profiles, the non-dimensional power parameter, S^*, and observe the non-dimensional control time, τ_c, from the x response plots.
4. The resultant cluster thrusts required, F_i, for the control system can be found from F and dimensionalizing according to Equation 4.12. The root-mean square power and propellant required can be found from Equations 4.15 and 4.16, respectively.

One of the more important results obtained from the attitude control study was that the root-mean-square power required to damp out a disturbance is proportional to the square of the spin rate,

as shown in Equation 4.15. However, the torus diameter, D_t, can be written in terms of the spin rate and the g-level to be provided (see Chapter 5) as

$$D_t = \frac{n_g\, g}{2 n_s^2}\left(\frac{60}{2\pi}\right)^2 \qquad (4.17)$$

where the spin rate n_s is now in revolutions per minute (rpm's). Inserting this equation into Equation 4.7 and converting the spin rate to rpm's yields

$$P_{rms} = \left(\frac{2\pi}{60}\right)^4 \frac{V_{ex}\, I_z\, n_s^4}{n_g\, g}\sqrt{\frac{S^*}{\tau_c}}\,. \qquad (4.18)$$

Equation 4.18 clearly shows the benefit of keeping the spin rate below 1 rpm (since for $n < 1$, $n^4 < n$) in order to decrease the power requirements for attitude control. This result was taken into account in human factors studies when the torus was designed (see Chapter 5).

4.4 CONCLUSION

This year, the work on the dynamics and control aspect of Project WISH focused on determining the attitude control system requirements. A methodology was established to perform attitude control studies for a wide variety of missions. This methodology, which is summarized above, was used to design the attitude control system for the two missions of the Emerald City. The results of these mission designs can be found in Chapter 6.

CHAPTER 5

HUMAN FACTORS

5.0 INTRODUCTION

Maintaining a habitable environment in space presents problems with varying degrees of difficulty depending upon its size and how long that environment is to be maintained. Something such as food supply, which is a relatively minor concern on a shuttle mission, becomes a problem that could dominate the design of a spacecraft with a crew of between 500 and 1000 and an operational lifetime of fifty years. Because of the magnitude of the Emerald City, human factors will play an important role in its design. This chapter covers three topics that relate directly to the design of the spacecraft: the life support system, the need for artificial gravity, and the problem of radiation.

5.1 LIFE SUPPORT SYSTEM

Last year, the different functions of life support were approached using mechanical systems[2]. Separate systems were used for air revitalization, temperature and humidity control, waste management, and so forth. However, there are two major faults with using mechanical systems. Because of the three year plus mission times expected for Emerald City, all of these systems would have to run (unrealistically) with one-hundred percent efficiency. This is necessary to prevent compounds such as air or water from being recombined into products that cannot be recycled. More importantly

though, is that since these systems are mechanical, they are subject to breakdown and will eventually need spare parts. Storing these parts or making provisions for their manufacture would waste valuable space and add extra mass to the ship. For these reasons, another concept was explored for the primary life support system: biospheres.

A biosphere is a totally enclosed ecological system, with energy as its only input. The collective biomass of the earth is referred to as Biosphere I. Since all air, water, and waste is recycled internally, life can be sustained indefinitely. Clair Folsome created the first human-made biosphere inside a lab flask[10]. The flask contains only simple organisms, but it has been alive since it was sealed in 1968. What is proposed for Project WISH is that a scaled and more efficient version of Biosphere I be used as the primary life support system. Research is already being done to create biospheres large enough for human occupation. The Biosphere II project[10] in Oracle, Arizona has constructed an environmental facility sealed off from the atmosphere that uses its land and water four and ten times as efficiently as Biosphere I[11]. Total recycling of all atmospheric carbon dioxide takes place in half a day instead of eight years. The 150,000 cubic meter facility is designed as an environment for eight people who were recently sealed inside for a two year study. This figures to roughly 19,000 cubic meters per person. Even though this number would probably shrink if research were devoted to reducing the volume needed per person, it was used for sizing the crew section

to allow extra space for manufacturing, food processing, or other needs that have not yet been considered. The equation for the volume required in the torus section was then given by

$$V_{torus} = 19000N \tag{5.1}$$

where N is the number of crew members on the Emerald City. Research is currently aimed at understanding more about Biosphere I and to possibly extend man made biospheres onto other planets. It was believed that the concept of biospheres could legitimately be considered for a spacecraft with the life expectancy and magnitude of Project WISH.

5.2 ARTIFICIAL GRAVITY AND SHIP DESIGN

Of the little information known about long term effects of zero-g on the body, none of it is very positive. Without gravity, bones decalcify and muscles weaken and atrophy. A condition that could be described as "space anemia" also occurs[12]. This is due to a drop in the production of red blood cells. This otherwise harmless condition could pose a serious health problem if an astronaut were injured or became ill. To counter these effects, the crew section of Emerald City was designed as a torus rotating about a central hub to provide some level of artificial gravity. Even though providing 1 g would eliminate these effects, considerable power and mass savings can be made if an acceptable lower g-limit can be found. The relationship between rotation rate, distance from the spinning axis, and g-level is shown in Figure 5.1. Since the volume requirement (from Section 5.1) and shape for

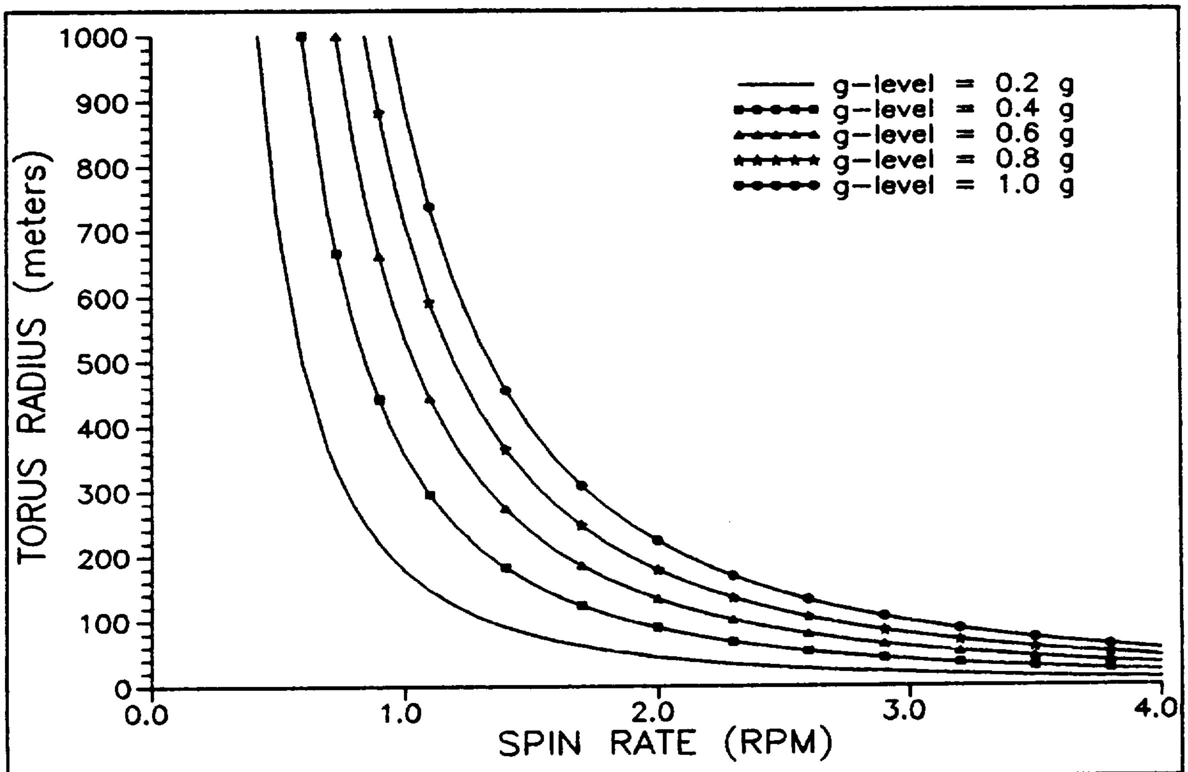

Figure 5.1 - Radius vs. Spin Rate for Different Gravity Levels

the crew section is known, the actual geometry is determined from the g-level and spin rate desired. Several possible geometries are listed in Table 5.1. Selection of the major radius of the torus, R (see Figure 6.1), is dependant on the constraints set for g-level and spin rate. One revolution per minute is used as an upper limit on spin rate because the power needed to maintain the attitude of the Emerald City is directly proportional to spin rate to the fourth power. This is discussed in greater detail in Chapter 4. Another reason behind this limit is human endurance. Most humans cannot endure extended periods of rotation at rates higher than 1 rpm[12]. Although there is no universal agreement on this limit, 1 rpm was chosen as it was the most conservative limit found. To

conserve mass, providing a g-level of only .8 g in the torus was considered. This lower gravity would hopefully have only minor effects on the crew. Even lower gravity levels could be used, which would reduce the mass further, but more information on the effects of long term exposure to lower gravity levels is needed before a lower g-limit can be set.

Table 5.1 - Possible Torus Geometries

Crew of 1000						
major radius (m)	minor radius (m)	%change in G top-bot	outer radius (m)	n for .8g (rpm)	n for .7g (rpm)	n for .6g (rpm)
500	44.2	17.7	544	1.15	1.07	0.99
540	42.5	15.7	582	1.11	1.04	0.96
580	41.0	14.1	621	1.07	1.00	0.93
620	39.6	12.8	660	1.04	0.97	0.90
660	38.4	11.6	698	1.01	0.95	0.88
700	37.3	10.7	737	0.99	0.92	0.85
Crew of 500						
major radius (m)	minor radius (m)	%change in G top-bot	outer radius (m)	n for .8g (rpm)	n for .7g (rpm)	n for .6g (rpm)
500	31.2	12.5	531	1.16	1.09	1.01
540	30.0	11.1	570	1.12	1.05	0.97
580	29.0	10.0	609	1.08	1.01	0.94
620	28.0	9.0	648	1.05	0.98	0.91
660	27.2	8.2	687	1.02	0.95	0.88
700	26.4	7.5	726	0.99	0.93	0.86

Another factor that contributes to the mass of the torus is the fact that it must hold an atmosphere. If a stressed skin is used, the necessary skin thickness and structural mass (M_{ss}) of a torus, with major radius R and minor radius r, is found using the formulas shown on the top of the next page[13]. The first two formulas find the skin thicknesses needed to withstand the principal stresses on the surface of the torus. Generally, t_{hoop} is greater than $t_{meridional}$, so t_{hoop} is used to find the structural mass. Structural mass is calculated for several configurations later in this chapter using the working stress (σ_w) and density (ρ) of

$$t_{meridional} = \frac{P_{atm} r}{\sigma_w}$$

$$t_{hoop} = \frac{\frac{P_{atm}}{2}\frac{r}{R} + \frac{P_g}{\pi}}{\sigma_w - \rho R} R \tag{5.2}$$

$$M_{ss} = 4\pi^2 r R t_{hoop} \rho$$

aluminum. P_g is the pressure due to the load distribution inside the torus. One way to reduce the structural mass is to lower the atmospheric pressure (P_{atm}) inside. Doing this would also result in effects similar to living at higher altitudes on Earth: the coughing mechanism would be less effective, a normal speaking voice would not carry as far, boiling temperature would be lowered, etc. Since structural mass is relatively minor compared to the whole ship, a pressure of one atmosphere is used in the crew section for all calculations.

5.3 RADIATION AND SHIELDING

The principle risk of living in space is from radiation induced cancer. The National Council on Radiation Protection and Measurements[14] has recommended a career limit for whole body exposure with a lifetime three percent excess risk of fatal cancer. Figure 5.2 is based on age at the start of exposure with a ten year career assumed. This recommendation, however, is made not for exploratory missions, such as a Mars mission or those that would be undertaken by the Emerald City, but for multiple missions of approximately 90 days over a 10 year period. What is needed for

Project WISH is a limit for constant exposure. Table 5.2 shows the limits recommended for constant public and occupational exposure in the United States. The occupational annual exposure limit of 5 rem per year would be the limit used on the Emerald City.

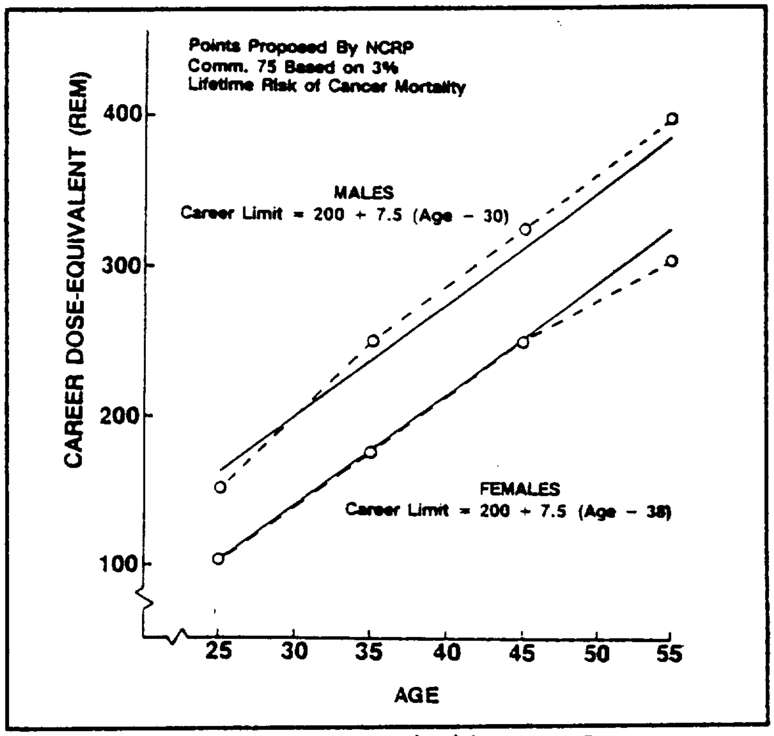

Figure 5.2 - Career Limit vs. Age

There are four major sources of radiation for Project WISH: galactic cosmic radiation, radiation from solar flare events, the propulsion system, and the power system. Of these four, only the last two can be reduced by moving the source away from the crew or by covering the source with a shield. Cosmic rays are present throughout the solar system at relatively constant intensities, decreasing only during solar flare activity. Unshielded dose rates from cosmic rays are estimated to be from 30 to 50 rem per

Table 5.2 - Continuous Dose Limits

Occupational	
Annual	5.0 rem
Lifetime (guideline)	Age*1.0 rem
Planned or special emergency	10.0 rem
Public	
Annual, continuous	0.1 rem
Annual, occasional	0.5 rem

year[15], while large solar flares are capable of delivering doses in excess of 1000 rem in only a week[16]. Since shielding these sources is impossible, the only way to protect from them is to shield the crew section. Radiation from the power system and the gaseous uranium core of the propulsion system can be attenuated by separating them from the crew and by partial shielding (see Figure 5.3). Unfortunately, the expanding plume of exhaust from the gas-

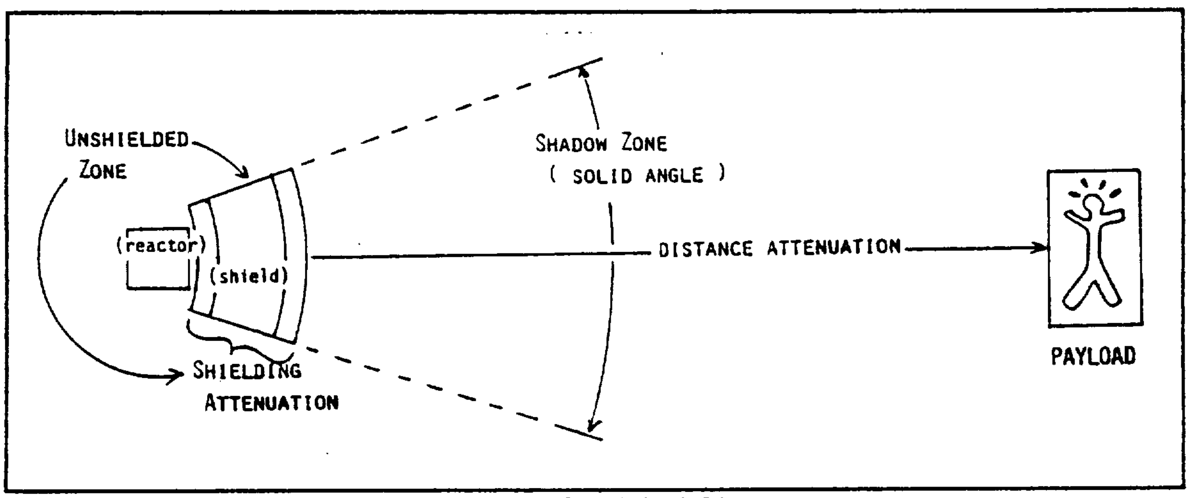

Figure 5.3 - Method of Partial Shielding

core rocket contains fission fragments that emit gamma radiation back towards the crew section. Increasing the distance between the crew and the nozzle exit will reduce the dose rate, but structural limits will only allow so much separation. The only other means of attenuation is to shield the crew section. The actual dose rate was calculated with the parameters of the gas-core engine from Chapter 3 using the formulas given by Charles Masser[17]. It is primarily dependant on the amount of separation between the crew and the engine, and the amount of time that fission fragments

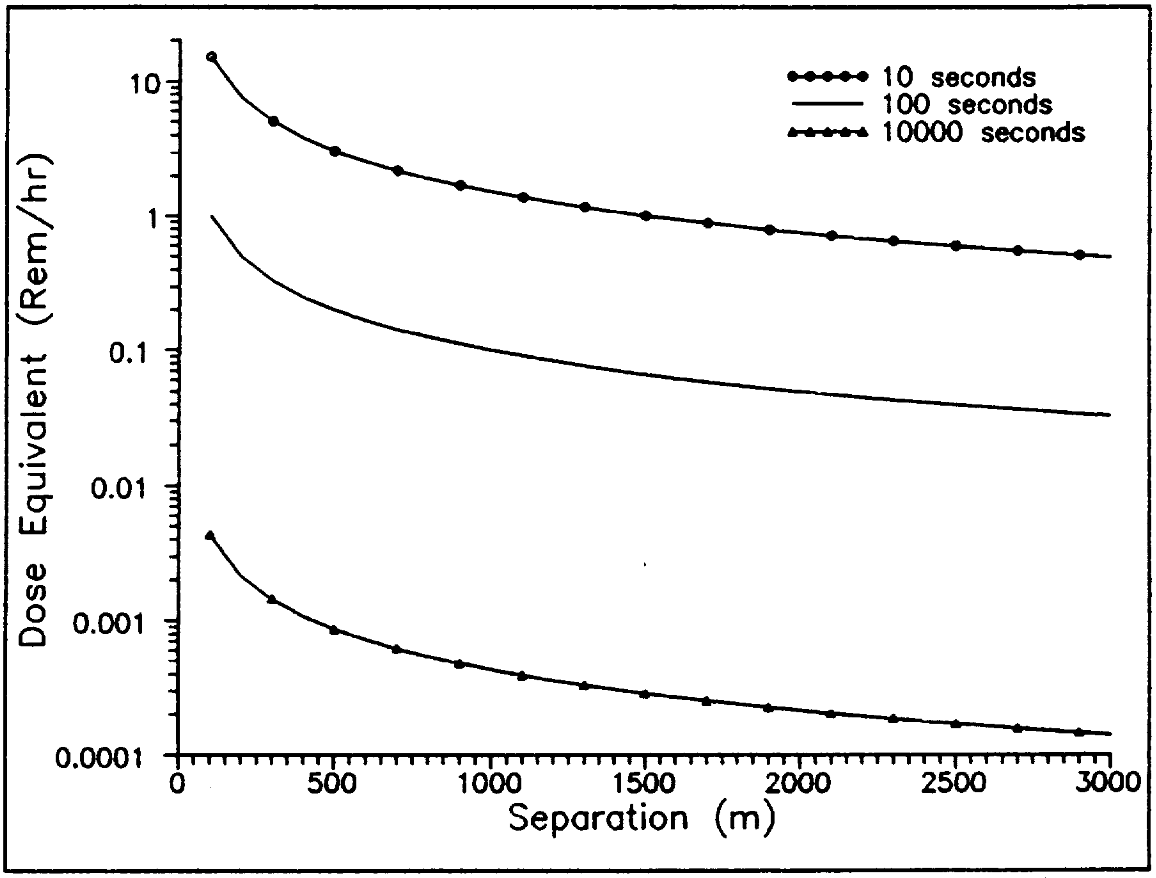

Figure 5.4 - Dose Rate vs. Separation for Varying Fission Fragment Retention Times

remain in the core before exiting the nozzle. Figure 5.4 shows this relationship for three different fission fragment retention times. The only estimate for the fission fragment retention time of the gas-core engine comes from Masser, who estimates it to be about 100 seconds. Note that the numbers in Figure 5.4 are only for one engine. It has been assumed that for the case of multiple engines, the dose rate will be additive. Although this assumption is probably not accurate, it is one that had to be made because of the limited knowledge of radiation by the members of this project.

Galactic cosmic radiation is considered to be the primary source of radiation because it is always present and cannot be

shielded at the source. No matter what their intensity, solar flares and engine exhaust will only be present for periods of about a week or two at a time. Because of this, the selection of a primary form of shielding was based on protecting against galactic cosmic radiation. The most common method of shielding is passive shielding. Passive shielding, known to be effective, simply puts a physical barrier between the source and the target. A more theoretical type of shielding is active shielding. It uses either an electromagnetic or electrostatic field to repel incoming radiation. A drawback common to both field types is that, depending on how much power they receive, they only shield radiations up to a certain energy level. Electrostatic fields were eliminated because a potential on the order of 3 GV is needed on the surface of the torus[18]. This potential would limit extra-vehicular activity and also generate lethal bremsstrahlung radiation fields inside the torus. An electromagnetic shield was rejected because higher energy radiation could become trapped inside the field. This would create something similar to the Van Allen belts around the Earth only much more dangerous considering the smaller distances involved. Another common failing of the shields is that they only stop charged particles. Gamma rays and neutrons, the more destructive types of radiation, are not affected.

With only passive shielding remaining, materials that could best attenuate radiation and still save mass were explored. Town-shend compares different materials and finds liquid hydrogen to be

very effective[19]. Liquid hydrogen is more effective than other materials because it contains no neutrons for the incident radiation to scatter upon collision. Table 5.3 shows that 100 grams per square centimeter will reduce the dose to below 5 rem per year. One hundred grams per square centimeter of liquid hydrogen corresponds to a shield thickness of 14 meters. The mass of this type of shield, as well as the structural mass, is found for four different torus configurations in Table 5.4. Notice that even if the crew size is cut in half, the masses for the torus do not significantly drop. This is because of the geometrical nature of a torus, a large volume change results in a relatively minor change in surface area.

5.4 CONCLUSION

Even though all the considerations in this chapter have not been solved, enough has been determined to create a pattern that can be used to make an initial design for the Emerald City. The results of research in the areas of biospherics, microgravity, radiation, and other fields not yet considered will set the limits that help finalize any design. The information presented in this chapter will be used for a preliminary design of the Emerald City for two particular missions, which may be found in Chapter 6.

Table 5.3 - Solar Minimum Galactic Cosmic Ray Depth Dose Equivalent in Tissue as a function of Particle Type and LH_2 Shield Thickness

thickness (gm/cm^2)	dose equivalent, rem/yr, from				
	neutrons	protons	alpha's	HZE	total dose
	skin dose equivalent (0 cm depth)				
0.0	0.0	9.4	6.7	101.6	117.7
3.0	0.2	6.6	2.7	31.8	41.3
10.0	0.6	7.8	1.5	6.3	16.2
25.0	0.8	8.1	0.4	0.4	9.7
50.0	0.7	6.6	0.1	<.1	7.4
75.0	0.6	4.8	<.1	<.1	5.4
100.0	0.4	3.3	<.1	<.1	3.8
	BFO dose equivalent (5 cm depth)				
0.0	1.4	8.0	2.9	43.0	61.1
3.0	1.8	8.8	2.2	21.2	34.1
10.0	1.9	9.6	1.2	4.4	17.2
25.0	1.7	9.4	0.1	0.3	11.7
50.0	1.3	7.4	<.1	<.1	8.7
75.0	0.9	5.3	<.1	<.1	6.2
100.0	0.7	3.6	<.1	<.1	4.3

Table 5.4 - Structural and Shield Mass Estimates

crew size	major radius	minor radius	surface area	shield mass	structural mass	skin thickness
	(m)	(m)	(m^2)	(kg)	(kg)	(cm)
1000	500	44.2	8.725e+05	8.725e+08	4.101e+07	1.741
1000	700	37.3	1.031e+06	1.031e+09	5.052e+07	1.815
500	520	30.6	6.282e+05	6.282e+08	2.407e+07	1.419
500	700	26.4	7.296e+05	7.296e+08	3.027e+07	1.537

CHAPTER 6

REPRESENTATIVE MISSION DESIGN

6.0 INTRODUCTION

Once the methodologies of the previous chapters have been formulated, two sample missions were examined in order to obtain some preliminary sizing and mass estimates for the Emerald City. A Saturn Envelope mission and the currently popular Earth-to-Mars mission were chosen as representative scenarios for this preliminary design. The design for these missions were determined by incorporating the orbital mechanics, propulsion, attitude control and human factors subsystems. Orbital mechanics (Chapter 2) was used to determine the ΔV's required for the selected missions. The propulsion system study (Chapter 3) was used to determine the number of engines and the total engine and propellant masses. These were then used to size the Emerald City in terms of volume of propellant, engine spacing and location, length of central pole, etc. Human factors (Chapter 5) was used to determine the sizing of the torus, and the attitude control study (Chapter 4) was used to calculate the control thruster requirements. With all of the work laid down in the previous chapters in mind, it is now possible to proceed with the design of both missions.

6.1 PROCEDURE

The procedure that was followed in the design of these missions is listed sequentially as follows.

1. Obtain the ΔV required for each mission from orbital mechanics.
2. Set values for the specific impulse, I_{sp}, and the thrust per engine, F_i, for the propulsion system. The mass of each engine, m_{wi}, is then found from the computer program of Appendix B.
3. Determine the torus mass, m_{torus}, from the masses of the biosphere, the shield, and the torus structure. These masses are dependant upon the number of crew members selected. The dry mass, m_{dry}, is then estimated by making allowances for the engine, cargo, and remaining subsystem masses.
4. Calculate the number of engines required and the total engine mass, m_w, for each mission.
5. Calculate the payload mass, m_l, for the missions. This mass is defined as $m_{dry} - m_{torus} - m_w$ and includes cargo as well as other systems that have yet to be sized (heat rejection, power, tankage, etc.).
6. Calculate the total initial mass, m_0, and the propellant mass, m_p. The propellant mass ratio, m_p / m_0, and the payload mass ratio, m_l / m_0, are then found.
7. Calculate the volume of propellant required for the mission, V_p. Then, by estimating a stem radius, r_p, the height of propellant, and hence the minimum stem length, h_p, can be found.

Neptune and Pluto cannot be reached at all in flight times of up to five years with this ΔV ceiling. It should be noted that many of the transfers to Saturn can be made with ΔV's considerably less than 50 km/s round trip, but for this mission, it was assumed that the Emerald City was in a poor orientation (ΔV-wise) with Saturn and that a three year flight time was essential. A possible motivation for such a mission might be that the Emerald City is required to support the first manned exploration of Saturn's moon Titan, and that the planetary alignments and limitations on the interplanetary transfer ship require a particular launch date from the Martian colony and arrival time at Saturn.

For this mission, the individual ΔV's and the powered-flight times required for the four separate burns were not considered, but only the total ΔV and powered-flight time. The number of engines was found based on these total values and a given total powered-flight time of 20 days, but it should be kept in mind that the powered-flight times would not be the same for all four burns, even if the ΔV required would be the same. This is because the mass of the Emerald City is smaller for each of the burns, and hence the longest powered-flight time would be the first one.

The results for the Saturn mission can be found in Tables 6.1. Some important values to note are the total initial mass, 4.16×10^9 kilograms, the number of engines, 172, the torus radius, 700 meters, and the height of the pole section, 1270 meters. The payload mass ratio for this mission was found to be 0.083, or 8.3% of the total initial mass is payload. It must be remembered,

however, that the payload mass as defined here is the mass of the cargo plus the mass for any other systems that were not considered during Phase II (power systems, thermal rejection systems, tankage mass, etc.).

6.2.2 Earth-to-Mars Mission

The second mission that was examined was a transfer from 1 AU to Mars. This mission represents a more common mission that the Emerald City might be required to undertake, perhaps carrying new colonists and supplies that had been shuttled to the Emerald City, just outside the Earth's sphere of influence, to the ever expanding Martian colonies. For this mission, the ΔV required was broken down into the departure and arrival ΔV's. The mission was scheduled to leave Earth on November 7, 2050 and the flight time was determined by minimizing the ΔV for this launch date using the MULIMP program. The ΔV's were found to be 5.1 km/s and 7.5 km/s for a total ΔV of 12.6 km/s, and the flight time was found to be 1.52 years. The first powered-flight time was set at 5 days and the number of engines required was determined. Then, keeping this number of engines constant, the second powered-flight time was found. From Table 6.1, it can be seen that the results of this were found to be 33 engines, a 5 day initial powered-flight time, and a 6.53 day terminal powered-flight time. Some of the other important design values for this mission are the initial mass, 1.295×10^9 kg, the torus radius, 700 meters, and the pole height, 569 meters.

Table 6.1 - Summary of Design Variables for Sample Missions

	Saturn Mission	Earth to Mars
People	1000	500
ΔV_{total} (km/s)	50	12.6
ΔV_1 (km/s)	-	5.1
ΔV_2 (km/s)	-	7.5
F_i (Newtons)	4.4×10^5	4.4×10^5
I_{sp} (seconds)	5000	5000
$t_{pr,total}$ (days)	20	11.53
$t_{pr,1}$ (days)	-	5
$t_{pr,2}$ (days)	-	6.53
number of engines	172	33
m_{dry} (kg)	1.5×10^9	1×10^9
m_{torus} (kg)	1.111×10^9	7.746×10^8
m_w (kg)	4.4376×10^7	8.514×10^6
m_l (kg)	3.457×10^8	2.165×10^8
m_p (kg)	2.658×10^9	2.947×10^8
m_o (kg)	4.16×10^9	1.295×10^9
m_l / m_o	0.083	0.167
m_p / m_o	0.639	0.228
V_{LH2} (m^3)	3.742×10^7	4.151×10^6
r_{pole} (m)	100	50
h_p (m)	1200	528
h (m)	1270	569
R (m)	700	700
r_{min} (m)	37	26
I_x (kg m^2)	5.6144×10^{14}	3.8061×10^{14}
I_y, I_z (kg m^2)	9.9568×10^{14}	2.1731×10^{14}
r (I_x / I_y)	0.563876	1.75154
max/min g-levels	0.8/0.72	0.8/0.74
n (spin rate, rpm)	0.99	0.99

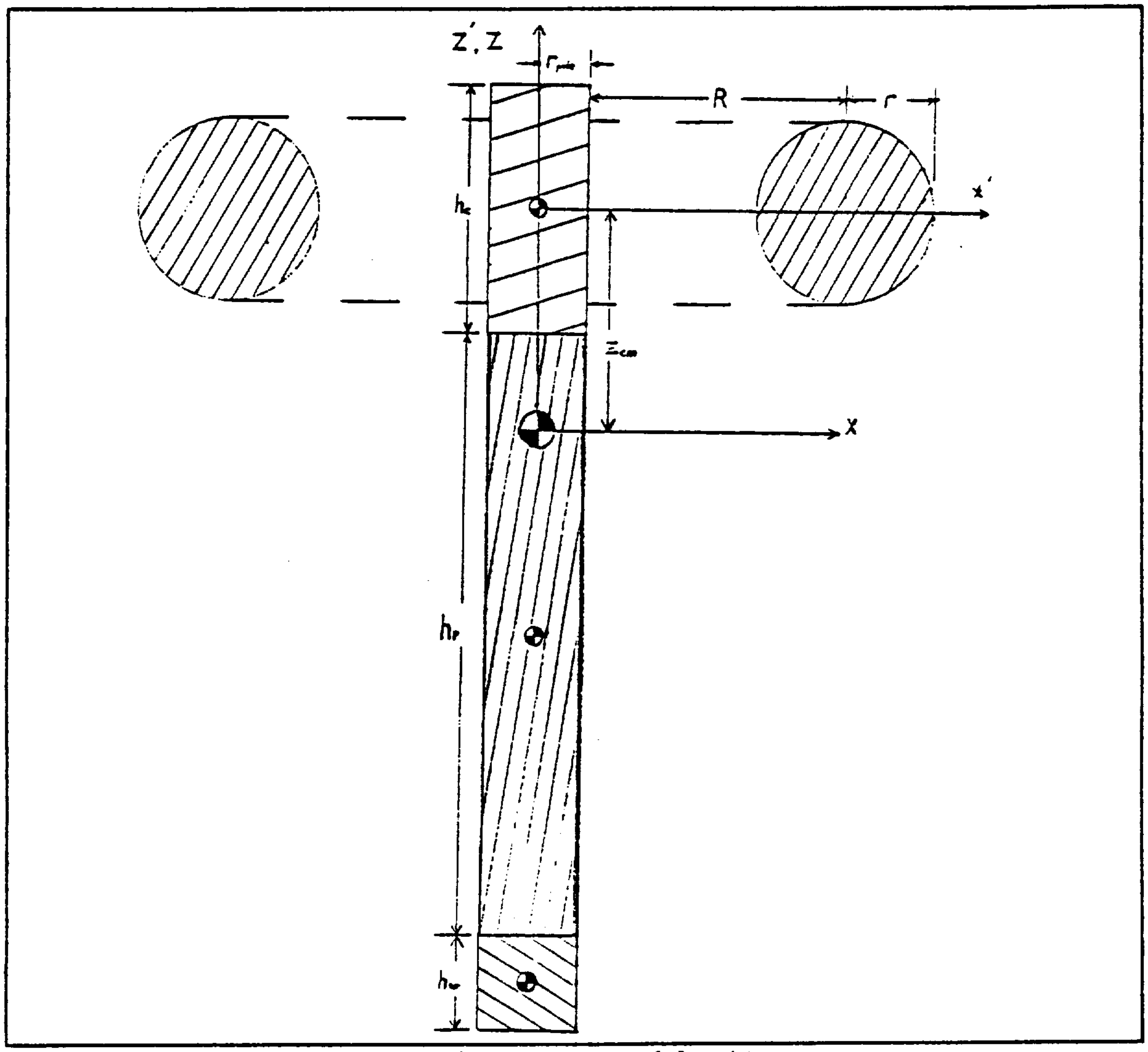

Figure 6.1 - Schematic of the Emerald City

6.2.3 Attitude Control System Requirements

An attitude control system was designed for the two particular missions examined for a sample initial disturbance and a sample thruster configuration. Using the methodology developed in Chapter 4, the root-mean-square power, propellant mass, maximum thrust, and control time was determined, as may be seen in Table 6.2. Also, the state response, control torque profile, and thrust profile for

the Saturn Envelope mission configuration are shown in Figures 6.2-6.4 for the sample thruster configuration shown in Figure 6.5.

Table 6.2 - Attitude Control System Requirements

I_z / I_x	0.56	1.75
Initial Disturbance	$[\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2] =$	$[.1 \quad .1 \quad .1n \quad .1n]$
P_{rms} (watts)	6.176×10^{12}	7.86×10^{11}
$m_{p,control}$ (kg)	2.105×10^{7}	3.458×10^{6}
$F_{control,max}$ (N)	2.988×10^{9}	1.911×10^{9}
$t_{control}$ (s)	~ 100	~ 100

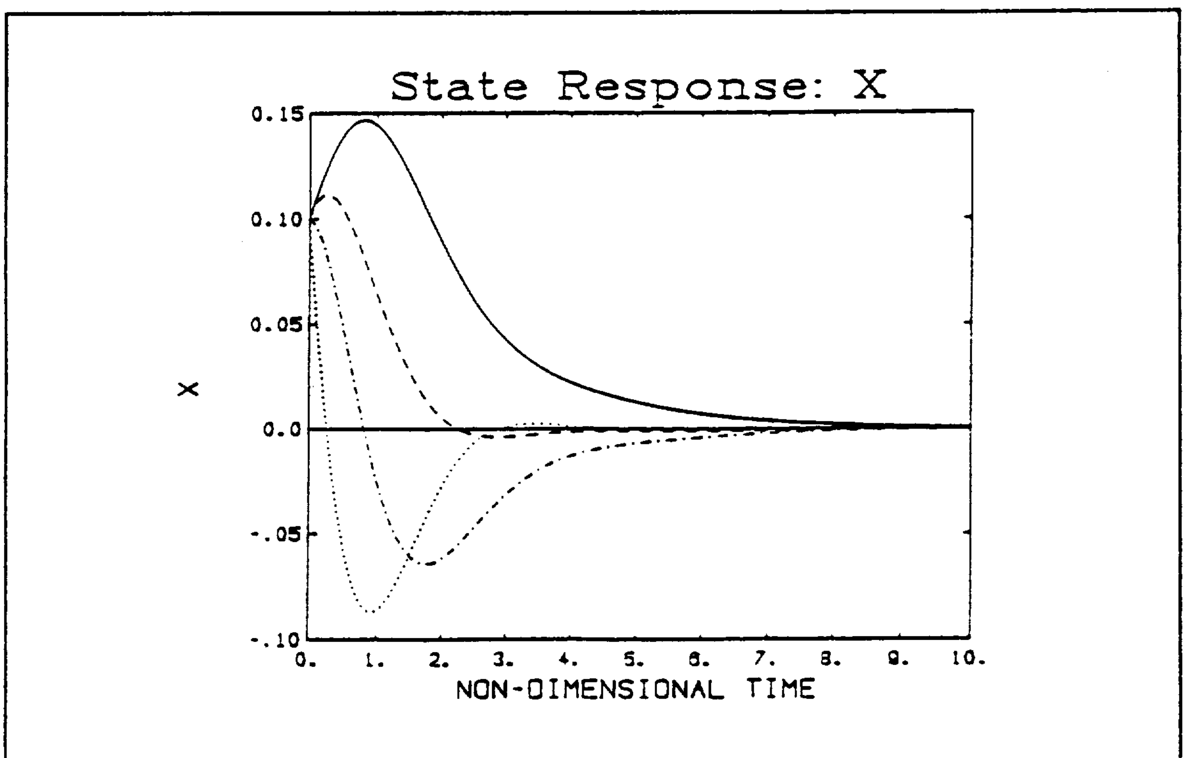

Figure 6.2 - Non-Dimensional State Response (r = 0.5)

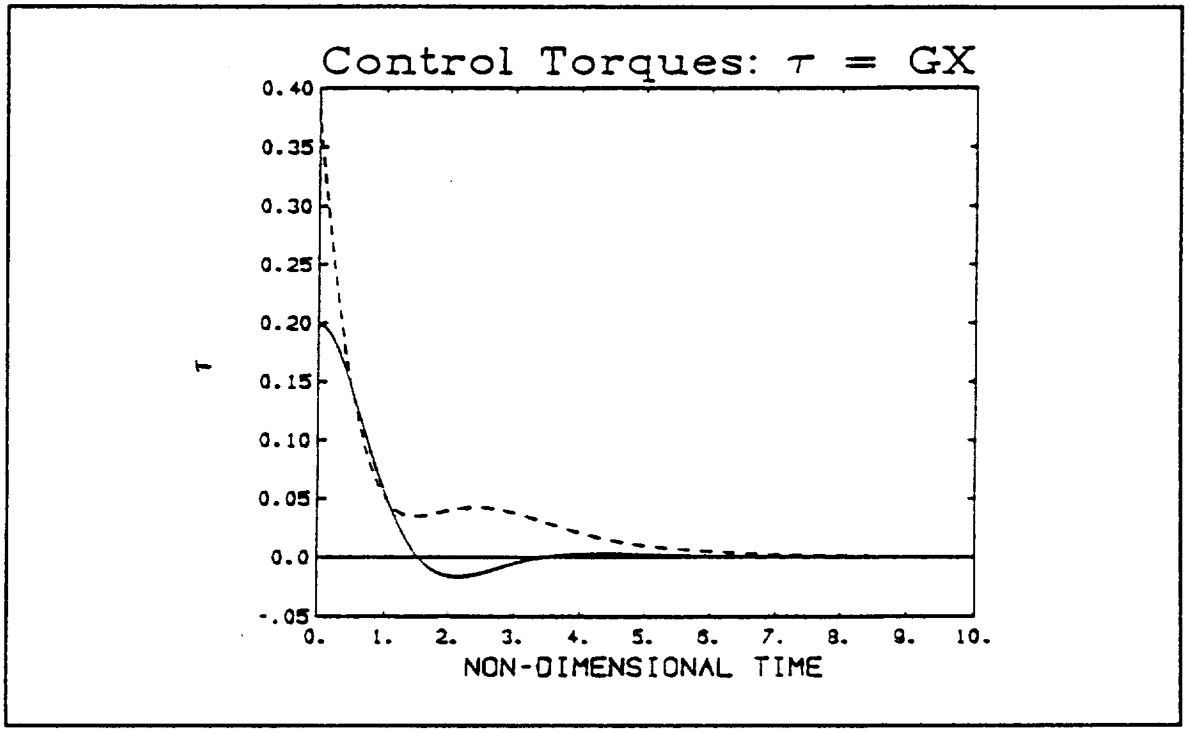

Figure 6.3 - Control Torque Profile (r = 0.5)

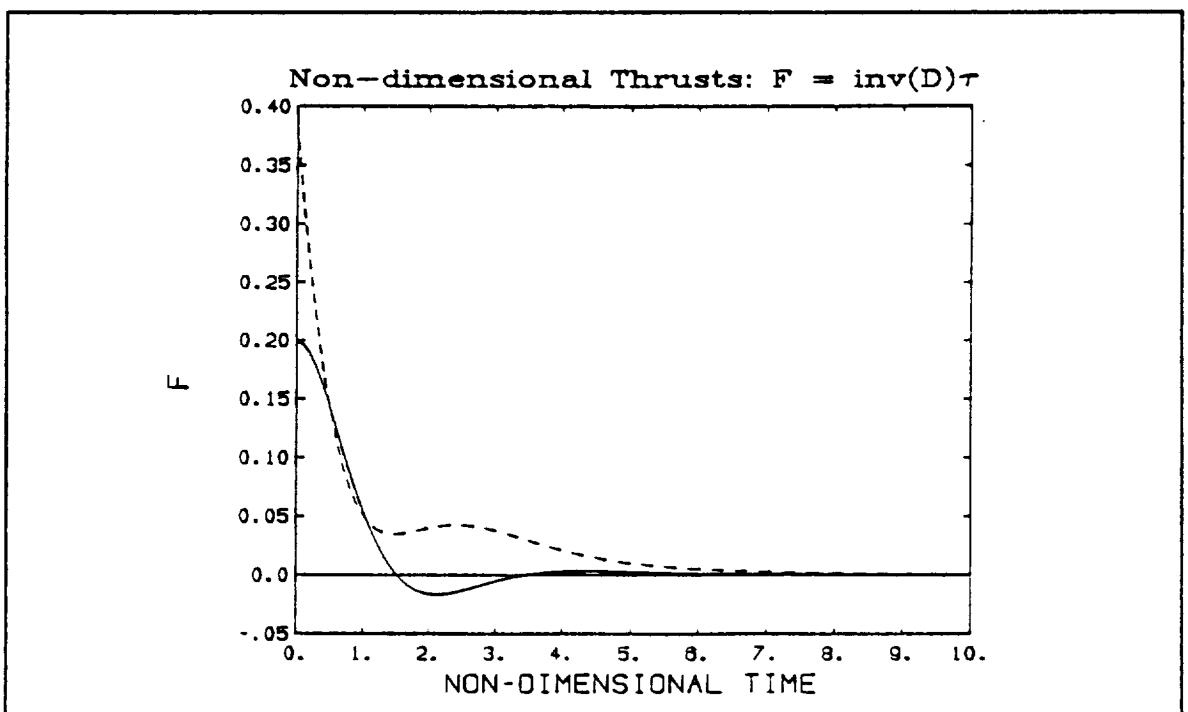

Figure 6.4 - Non-dimensional Thrust Profile (r = 0.5)

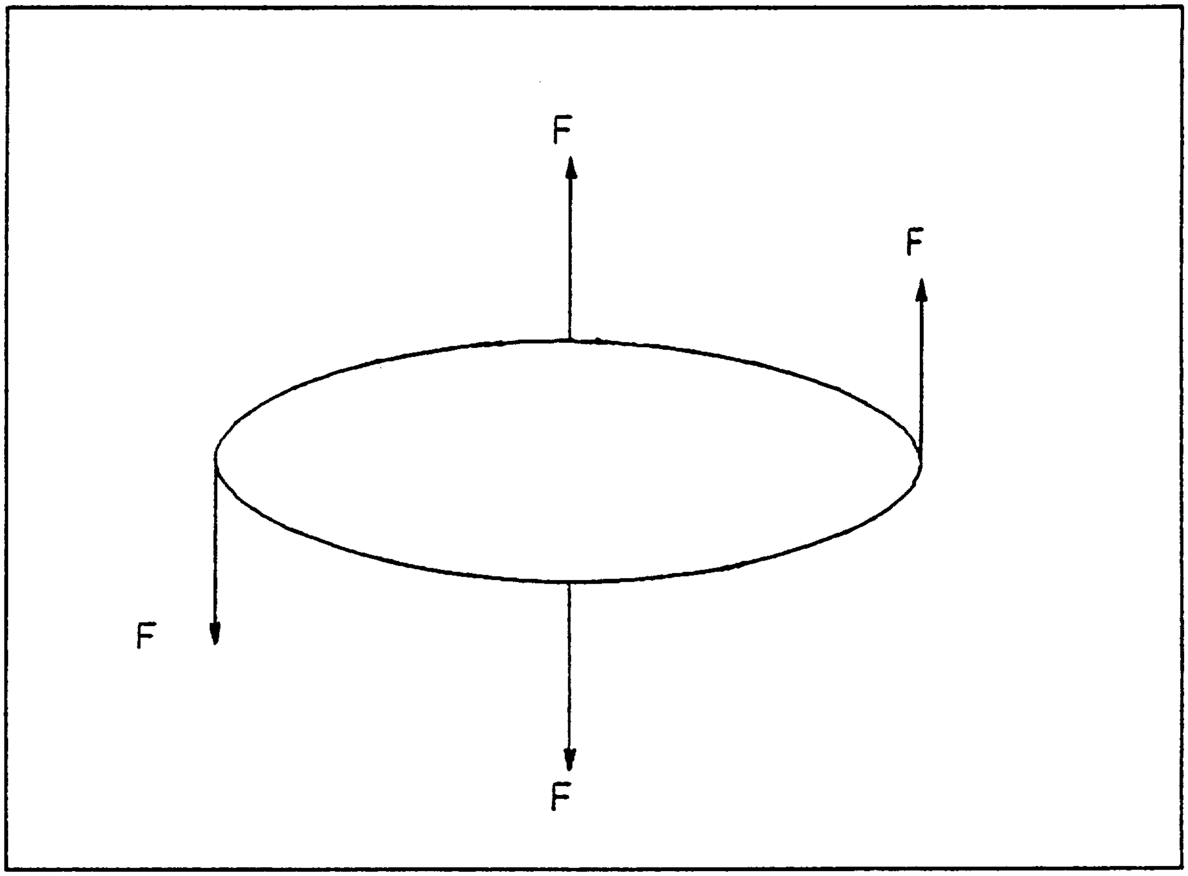

Figure 6.5 - Sample Thruster Configuration

6.3 CONCLUSION

In this chapter, two particular missions were examined for the Emerald City. However, using the methodology established here, a wide variety of missions may be looked at and the configuration for the Emerald City may be determined. In the future, different mission parameters, such as crew size, ΔV requirements, sample initial disturbances, attitude thruster configurations, etc..., may be examined. It is felt that this methodology is the main accomplishment of the Phase II design of Project WISH.

CHAPTER 7

CONCLUSION

7.0 PROJECT SUMMARY

Phase II of Project WISH was able to accomplish a great deal towards the design of the Emerald City. In orbital mechanics, a nominal orbit of 4 AU's was selected, and the operational envelope for the Emerald City was determined. An open-cycle gas-core nuclear thermal propulsion system was chosen as the main propulsion unit, and a detailed study was conducted on this type of engine. For attitude control, a methodology was established to determine the thruster requirements needed to damp out a disturbance experienced by the Emerald City. Human factors looked at the problems of life support, artificial gravity, and radiation shielding. Biospheres were examined as the primary method to sustain life, while the torus size requirements were determined for the different levels of gravity that the crew would need. Radiation shielding proved to be one of the major design problems encountered, and mass estimates were obtained for shielding from cosmic radiation. The ultimate result of the Phase II study was the generation of Tables 6.1 and 6.2. These tables incorporated information from all of the subsystems studied, and provide data on the estimated size and mass requirements of the Emerald City. By examining these figures, one can realize the magnitude of a ship such as the Emerald City.

concessions to the unknown, we had best frankly recognize that irrational faith in the value of a new capability often plays an essential part in the early evolution of a new concept. Certainly, at that point, the lack of a recognized application is a demonstrably unsound basis for the concept's rejection."

Table 7.1 - Envisioned Time Line for Project WISH

1989	- President Bush announces Mars Initiative to reach the Red Planet in 30 years.
1991	- NASA presents a program to send sensing probes throughout the solar system. Projects include the return to the inner solar system, exploration of the asteroid belt, and further missions to the outer planets.
1998	- Space Station Freedom becomes operational.
2001	- Heavy Lift Launch Vehicle makes its maiden flight.
2004	- Construction begins on a near-geosynchronous Earth orbiting space station.
2009	- U.S. returns to the moon.
2011	- Construction begins on the Moon Base. - Maiden flight of National Aerospace Plane.
2015	- Moon Base becomes fully operational.
2019	- First manned mission to Mars.
2023	- First living modules constructed on Mars. - Construction begins on Reusable Interplanetary Ships (R.I.S.'s) for carrying personnel and cargo.
2028	- Mars Base becomes fully operational.
2040	- Implementation of Project WISH.
2050	- The Emerald City becomes operational.

APPENDIX A

EQUATIONS FOR ΔV MINIMIZATION PROBLEM

The objective function given for the ΔV minimization problem set up in Chapter 2 was given by Equation 2.4

$$\Delta V_T = \Delta V_1 + \Delta V_2 = \vec{V}_1 - \vec{V}_o + \vec{V}_f - \vec{V}_2 \tag{2.4}$$

where ΔV_1 and ΔV_2 are the velocity changes along the thrust vector direction, which will be kept constant during each powered-flight phase. The constraints were given in Chapter 2 as

1. $t_{ff} + t_{pr1} + t_{pr2} < TOF_{max}$
2. $a_0 < a_{0,max}$
3. $t_{pr1} + t_{pr2} < t_{pr,max}$
4. $x_{f,EC} = x_{f,TO}$.
5. a_0, t_{pr1}, t_{pr2}, $t_{ff} > 0$.

Thus, the problem is to determine the relationships between these values and the design variables of the problem.

Since a circular nominal orbit and target planet orbit has been assumed, v_0 and v_f are given by the formulas for circular orbital velocity,

$$v_0 = \sqrt{\frac{\mu}{r_0}} \tag{A.1}$$

$$v_f = \sqrt{\frac{\mu}{r_f}} \tag{A.2}$$

which are known for a given nominal orbit and target planet. However, finding the velocities v_1, the velocity at the end of the first powered-flight phase, and v_2, the velocity at the beginning of the second powered-flight phase, is more involved. To find the equations for these velocities, the kinematic relationship

$$dv = a \cdot dt \tag{A.3}$$

was integrated along the direction of thrust, with an acceleration term in this direction given by

$$a(t) = \frac{a_o}{1 - \dfrac{a_o t}{I_{sp} g}}. \tag{A.4}$$

This term arises from the fact that as the Emerald City is burning with a constant thrust, it is expending mass, and thus the acceleration will increase with time according to Equation A.4. Inserting Equation A.4 into Equation A.3 and performing the integration yields

$$v = v_i - I_{sp} g \ln(1 - \frac{a_o t_{pr}}{I_{sp} g}), \tag{A.5}$$

which becomes

$$\Delta V_1 = v_{1F} - v_{oF} = - I_{sp} g \ln(1 - \frac{a_o t_{pr1}}{I_{sp} g}) \tag{A.6}$$

for the first powered-flight portion of the transfer. Similarly, for the second powered-flight portion of the transfer, the velocity change is given by

$$\Delta V_2 = v_{fF} - v_{2F} = -I_{sp}g\,\ln\left(1 - \frac{a_{o2}\,t_{pr2}}{I_{sp}g}\right)$$

where (A.7)

$$a_{o2} = \frac{a_o}{1 - \dfrac{a_o\,t_{pr1}}{I_{sp}g}}.$$

The subscript F in Equations A.6 and A.7 denotes the components of the respective velocity vectors projected along the thrust direction (it is assumed that the thrust angles, $\beta_{1,2}$, will not be changed during a powered-flight phase). It can easily be seen that by using Equations A.6 and A.7, the objective function of Equation 2.4 can be written as

$$\Delta V_T = -I_{sp}g\left[\ln\left(1 - \frac{a_o t_{pr1}}{I_{sp}g}\right) + \ln\left(1 - \frac{a_{o2} t_{pr2}}{I_{sp}g}\right)\right]. \quad \text{(A.8)}$$

where a_{02} is not an independent variable, but given in Equation A.7.

The application of the constraint functions needed many more equations and relationships. First, continuing in the same manner as was used to find Equation A.5, the relationship

$$dx = v\,dt \quad \text{(A.9)}$$

was integrated with the results from Equation A.5 inserted for v, and the result was found to be

$$x_{1,f} = x_{o,2} + v_{o,2}t_{pr1,2} + \frac{(I_{sp}g)^2}{a_{o,o2}}\Phi_{1,2}$$

where (A.10)

$$\Phi_{1,2} = \left(1 - \frac{a_{o,o2}\,t_{pr1,2}}{I_{sp}\,g}\right)\left[\ln\left(1 - \left(\frac{a_{o,o2}t_{pr1,2}}{I_{sp}\,g}\right) - 1\right] + 1.$$

Equation A.10 yields the position along the direction of the thrust vector and the subscript 1,f means at position 1 or f, etc. Thus, armed with Equations A.5, A.10, and some fundamental relationships from astrodynamics, the constraints can be related to the design variables of the problem in the following manner. First, the inertial X- and Y-components of the velocity and radius vectors at the end of the first powered-flight phase can be found by applying coordinate rotation matrices to Equations A.5 and A.10. These equations are given by

$$v_{1x} = -\sqrt{\frac{\mu}{r_o}}\sin\theta_o - I_{sp}g\sin(\beta_1 - \theta_o)\ln(1 - \frac{a_o t_{pr1}}{I_{sp}g})$$
$$v_{1y} = \sqrt{\frac{\mu}{r_o}}\cos\theta_o - I_{sp}g\cos(\beta_1 - \theta_o)\ln(1 - \frac{a_o t_{pr1}}{I_{sp}g}) \qquad \text{(A.11)}$$
$$v_1 = \sqrt{v_{1x}^2 + v_{1y}^2}$$

and

$$r_{1x} = r_o\cos\theta_o - \sqrt{\frac{\mu}{r_o}}\,t_{pr1}\sin\theta_o + \frac{(I_{sp}g)^2}{a_o}\sin(\beta_1 - \theta_o)\,\Phi$$
$$r_{1y} = r_o\sin\theta_o + \sqrt{\frac{\mu}{r_o}}\,t_{pr1}\cos\theta_o + \frac{(I_{sp}g)^2}{a_o}\cos(\beta_1 - \theta_o)\,\Phi \qquad \text{(A.12)}$$
$$\Phi = (1 - \frac{a_o t_{pr1}}{I_{sp}g})[\ln(1 - \frac{a_o t_{pr1}}{I_{sp}g}) - 1] + 1$$
$$r_1 = \sqrt{r_{1x}^2 + r_{1y}^2}$$

where the known variables are r_0 and I_{sp} and the design variables are θ_0, t_{pr1}, a_0, and β_1. Once the velocity and position vectors at the initial point of the free-flight trajectory (point 1) are known, the transfer orbit is defined by the following equations:

$$e_x = \frac{1}{\mu}[(v_1^2 - \frac{\mu}{r_1})r_{1x} - (r_{1x}\,v_{1x} + r_{1y}\,v_{1y})\,v_{1x}]$$

$$e_y = \frac{1}{\mu}[(v_1^2 - \frac{\mu}{r_1})r_{1y} - (r_{1x}\,v_{1x} + r_{1y}\,v_{1y})\,v_{1y}] \qquad \text{(A.13)}$$

$$e = \sqrt{e_x^2 + e_y^2}$$

$$\gamma_o = \cos^{-1}(\frac{e_x}{e}) \quad (quadrant) \qquad \text{(A.14)}$$

$$\mathrm{E} = \frac{v_1^2}{2} - \frac{\mu}{r_1} \qquad \text{(A.15)}$$

$$a = -\frac{\mu}{2\mathrm{E}} \qquad \text{(A.16)}$$

$$\gamma_1 = \cos^{-1}(\frac{a(1 - e^2) - r_1}{e\,r_1}) \quad (quadrant) \qquad \text{(A.17)}$$

$$h = \sqrt{a(1 - e^2)\mu} \qquad \text{(A.18)}$$

In the above equations, e is the eccentricity of the transfer orbit, τ_0 is the direction of the eccentricity vector, E is the specific energy of the transfer orbit, a is the semi-major axis of the transfer orbit, τ_1 is the true anomaly at the end of the first powered-flight time, and h is the angular momentum of the transfer orbit. Once the transfer orbit is defined through these equations, the variables at the terminal free-flight point (point 2) may be calculated. The design variable that was designated was γ_2, which is the true anomaly at point 2. This then defines the free-flight time of the transfer according to

$$t_{ff} = \sqrt{\frac{a^3}{\mu}}\ [2\pi k + (E_2 - e\sin E_2) - (E_1 - e\sin E_1)] \qquad \text{(A.19)}$$

where E is the eccentric anomaly, defined by

$$E_n = \cos^{-1}\left(\frac{e + \cos\gamma_n}{1 + e\cos\gamma_n}\right) \qquad \text{(A.20)}$$

and k is the number of times the periapsis is passed during the transfer. The rest of the values at point 2 are given by

$$r_2 = \frac{a(1 - e^2)}{1 + e\cos\gamma_2} \qquad \text{(A.21)}$$

$$v_2 = \sqrt{2\left(\mathbf{E} + \frac{\mu}{r_2}\right)} \qquad \text{(A.22)}$$

$$\theta_2 = \left[\frac{\gamma_o + \gamma_2}{360^\circ} - INTEGER\left(\frac{\gamma_o - \gamma_2}{360^\circ}\right)\right]\cdot 360^\circ \qquad \text{(A.23)}$$

$$\phi_2 = \cos^{-1}\left(\frac{h}{r_2 v_2}\right) \quad (quadrant) \qquad \text{(A.24)}$$

$$\begin{aligned} r_{2x} &= r_2\cos\theta_2 \\ r_{2y} &= r_2\sin\theta_2 \end{aligned} \qquad \text{(A.25)}$$

$$\begin{aligned} v_{2x} &= -v_2\sin(\theta_2 - \phi_2) \\ v_{2y} &= v_2\cos(\theta_2 - \phi_2)\,. \end{aligned} \qquad \text{(A.26)}$$

The equations governing the final powered-flight phase of the trajectory are found by applying coordinate rotation matrices to Equations A.5 and A.10, similar to the way that Equations A.11 and A.12 were obtained. These equations were found to be

$$\begin{aligned} v_{fx} &= -v_2\sin(\theta_2 - \phi_2) - I_{sp}g\sin(\beta_2 - \theta_2)\ln\left(1 - \frac{a_{o2}t_{pr2}}{I_{sp}g}\right) \\ v_{fy} &= v_2\cos(\theta_2 - \phi_2) - I_{sp}g\cos(\beta_2 - \theta_2)\ln\left(1 - \frac{a_{o2}t_{pr2}}{I_{sp}g}\right) \end{aligned} \tag{A.27}$$

and

$$\begin{aligned} r_{fx} &= r_2\cos\theta_2 - v_2t_{pr2}\sin(\theta_2 - \phi_2) + \frac{(I_{sp}g)^2}{a_{o2}}\sin(\beta_2 - \theta_2)\Phi_2 \\ r_{fy} &= r_2\sin\theta_2 + v_2t_{pr2}\cos(\theta_2 - \phi_2) + \frac{(I_{sp}g)^2}{a_{o2}}\cos(\beta_2 - \theta_2)\Phi_2 \\ \Phi_2 &= \left(1 - \frac{a_{o2}t_{pr2}}{I_{sp}g}\right)\left[\ln\left(1 - \frac{a_{o2}t_{pr2}}{I_{sp}g}\right) - 1\right] + 1. \end{aligned} \tag{A.28}$$

The above equations will play an important part in the determination of the constraints on the total time-of-flight and the rendezvous requirement. For the time-of-flight constraint, the constraint function is found by adding the two powered-flight times, which are design variables, to the free-flight time as found from Equation A.19. This value must then fall below the specified TOF_{max} (three years for most cases) for the constraint to be satisfied. For the rendezvous requirement, recall from Chapter 2 that the state vector at point i, x_i, was defined such that

$$\vec{x}_i = \{\vec{r}_i \quad \vec{v}_i\}. \tag{2.5}$$

The rendezvous constraint requires that $x_{f,EC} = x_{f,TO}$, which represents the Emerald City ending the transfer with the desired velocity and radius vectors at the target orbit. The state vector, $x_{f,EC}$, can be found from Equations A.27 and A.28, while the state vector, $x_{f,TO}$, can be determined from

$$\theta_{f,TO} = \theta_i + \sqrt{\frac{\mu}{r_p^3}}\, TOF \tag{A.29}$$

$$\begin{aligned} r_{x,TO} &= r_p \cos\theta_{f,TO} \\ r_{y,TO} &= r_p \sin\theta_{f,TO} \end{aligned} \tag{A.30}$$

$$\begin{aligned} v_{x,TO} &= -v_p \sin\theta_{f,TO} \\ v_{y,TO} &= v_p \cos\theta_{f,TO}. \end{aligned} \tag{A.33}$$

With the above formulation, care must be taken in the programming of the above equations to assure that the proper quadrant is used when inverse trigonometric functions are used.

APPENDIX B

ENGINE CHARACTERISTIC CALCULATIONS

A computer program was written to calculate the important engine parameters for an open-cycle gas-core nuclear engine. This program is listed below. The equations were either obtained from Ragsdale's paper or derived from basic considerations

```
*****************************************************************
      program gascore
*****************************************************************
* this program finds the parameters of a gas core rocket for a
* specific cavity diameter and Isp and prints the data to a file
* called engine.dat.                Enjoy.
*****************************************************************
      dimension f(5)
      open(unit=1,file="ENGINE.DAT")
      GRAV=9.80665
      PI=3.14159265
      ATM=1.01325E5
      UGC=8314
      THRUST=4.4E+5
      VUVC=.23
      write(6,10)
 10   format('1',5X,'ENTER SPECIFIC IMPULSE (Seconds)')
      read*,SPIMP
      write(6,40)
 40   format(5X,' ENTER REACTOR CAVITY DIAMETER (Meters)')
      read*,DIAM

**    Calculate critical mass of Uranium 233   **

      CD=3.3*DIAM
      CRITM=201.836-67.0167*CD+7.70629*CD**2-.158108*CD**3-
     &.0196678*CD**4+.00088141*CD**5

**    Calculate mass flow rate ratio        **

      DOTM=5.041871E-5*(VUVC**-10.799136)

**    Calculate hydrogen propellant temperature  **
      TH2=-294.461+4.36196*SPIMP+7.23731E-4*SPIMP**2-6.03953E-
     &7*SPIMP**3
      TH2=TH2+1.36261E-10*SPIMP**4-7.71588E-15*SPIMP**5

**    Calculate reactor cavity pressure        **
```

```
      PRES=(1066.57*CRITM*((THRUST*SPIMP)**.277)*(DOTM**.1012)/
     &(DIAM**3.277))**1.383

**    Calculate mass flow rate from nozzle    **

      PDOT=THRUST/(SPIMP*GRAV)

**    Calculate mass flow rate of hydrogen and uranium   **

      UDOT=PDOT/(DOTM+1)
      H2DOT=PDOT-UDOT

**    Calculate uranium temperature  **

      TU=140.06*((PRES*THRUST*SPIMP/DIAM)**.16)*(DOTM**.017)

**    Calculate pressure chamber volume and uranium volume   **

      VC=1.33333*PI*((DIAM/2)**3)
      VU=VUVC*VC

**    Calculate hydrogen ratio of specific heats in cavity   **

      GAMMAC=3.84502-.000336982*TH2+1.61689E-8*TH2**2-3.66763E-
     &13*TH2**3
      GAMMAC=GAMMAC+4.37181E-18*TH2**4-2.65532E-23*TH2**5+
     &6.49491E-29*TH2**6

**    Calculate hydrogen molecular mass in cavity   **

      H2MMC=2.32426-.000111528*TH2+2.53066E-9*TH2**2-2.49052E-
     &14*TH2**3
      H2MMC=H2MMC+8.94761E-20*TH2**4

**    Calculate throat conditions   **

      TTH=TH2/(1+((GAMMAC-1)/2))
      PTH=PRES/((1+(GAMMAC-1)/2)**(GAMMAC/(GAMMAC-1)))
      PTH=PTH/ATM
      GAMMAT=3.84502-.000336982*TTH+1.61689E-8*TTH**2-
     &3.66763E-13*TTH**3
      GAMMAT=GAMMAT+4.37181E-18*TTH**4-2.65532E-23*TTH**5+
     &6.49491E-29*TTH**6
      H2MMT=2.32426-.000111528*TTH+2.53066E-9*TTH**2-
     &2.49052E-14*TTH**3
      H2MMT=H2MMT+8.94761E-20*TTH**4
      ATH=PDOT*SQRT(TH2)/PRES
      ATH=ATH/SQRT(GAMMAT*H2MMT*(2/(GAMMAT+1))**
     &((GAMMAT+1)/(GAMMAT-1))/UGC)
      VTH=SQRT(GAMMAT*UGC*TTH/H2MMT)

**    Calculate exit mach number  **
```

```
      AMACH=3.00
 50   ANSW=(2*(1+((GAMMAC-1)/2)*AMACH**2)/(GAMMAC+1))
     &**((GAMMAC+1)/(GAMMAC-1))
      ANSW=ANSW/AMACH**2-9E4
      if(ANSW.gt.0) goto 60
      AMACH=AMACH+.05
      goto 50

**    Calculate exit conditions   **

 60   VE=SPIMP*GRAV
      TE=H2MMC*(VE/AMACH)**2/(UGC*GAMMAC)
      PE=PRES/((1+((GAMMAC-1)/2)*AMACH**2)**(GAMMAC/
     &(GAMMAC-1)))
      PE=PE/ATM
      AE=300*ATH
      GAMMAE=1.40018-3.60774E-6*TE-2.87154E-8*TE**2+5.90601E-
     &12*TE**3
      GAMMAE=GAMMAE-2.94162E-16*TE**4
      H2MME=-2.8375E-6*TE+2.01588

**    Calculate density of uranium   **

      RHOU=CRITM/VU

**    Calculate diameter of uranium  **

      DIAMU=2*(3*VU/4/PI)**(.3333333)

**    Calculate reactor power       **

      QR=((TU/950)**6.25)*DIAMU/PRES

**    Calculate thickness and mass of pressure shell **

      THPS=PRES*(DIAM+1.52)/(5.516E9)
      PSM=1333.33*PI*((DIAM+1.52+2*THPS)**3-(DIAM+1.52)**3)

**    Calculate mass of moderator, turbopump, nozzle, and
**    radiator  **

      EMM=191.67*PI*((DIAM+1.54)**3-DIAM**3)
      TPM=.000307181*H2DOT*((1.5*PRES)**(.66667))
      ENM=52861.26*THRUST/PRES
      PR=3E-6*SPIMP+4.4E-2
      RM=145*PR*QR

**    Calculate total engine mass   **

      ENGMASS=PSM+EMM+TPM+ENM+RM

**    Calculate engine specific mass and jet power  **
```

```
      ALPHA=204.96*ENGMASS/(SPIMP*THRUST)
      PJET=.5*THRUST*SPIMP*GRAV/1E6
**    Calculate exit velocity      **

      VEXIT=SPIMP*GRAV

**    Calculate thrust to weight ratio  **

      TWR=THRUST/(ENGMASS*GRAV)
      PRES=PRES/ATM
      write(1,70)
 70   format(' SPECIFIED VALUES:')
      write(1,80)SPIMP,THRUST,VUVC,DIAM
 80   format(1X,'Isp=',F8.2,' seconds',3X,'Thrust=',F11.2,'
     &Newtons',3X,'Vu/Vc=',F3.2,3X,'Dc=',F5.2,' Meters')
      write(1,90)
 90   format(//,1X,'CALCULATED ENGINE PARAMETERS:')
      write(1,100)
 100  format(/,1X,'ENGINE CAVITY CONDITIONS:')
      write(1,110)TH2,TU
 110  format(1X,'T(H2)=',F8.2,' Kelvin',19X,'T(U233)=',F9.2,'
     &Kelvin')
      write(1,120)PRES,CRITM
 120  format(1X,'Cavity Pressure=',F7.2,' Atm',13X,
     &'Critical Mass U233=',F6.2,' Kg')
      write(1,130)VC,VU
 130  format(1X,'Cavity Volume=',F5.2,' Cubic
     &Meters',8X,'Uranium
     &Volume=',F5.2,' Cubic Meters')
      write(1,140)DIAMU,RHOU
 140  format(1X,'Uranium Diameter=',F4.2,' Meters',12X,'Uranium
     &Density=',F6.2,' Kg/Cubic Meter')
      write(1,150)H2MMC,GAMMAC
 150  format(1X,'Hydrogen Molecular
     &Mass(Cavity)=',F5.3,2X,'Gamma(Cavity)=',F5.3)
      write(1,160)
 160  format(/,1X,'MASS FLOW RATES:')
      write(1,170)DOTM,PDOT
 170  format(1X,'H2-To-U233 Mass Flow Ratio=',F7.2,6X,
     &'Propellant Mass Flow Rate=',F6.2,' Kg/sec')
      write(1,180)H2DOT,UDOT
 180  format(1X,'H2 Mass Flow Rate= ',F7.3,' Kg/sec',6X,' U233
     &Mass Flow Rate=',F6.3,' Kg/sec')
      write(1,190)
 190  format(/,1X,'ENGINE DIMENSIONS:')
      write(1,200)THPS,PSM
 200  format(1X,'Pressure Shell Thickness=',F5.3,'
     &Meters',3X,'Pressure Shell Mass=',f8.2,' Kg')
      write(1,210)EMM
 210  format(1X,'Moderator Thickness= 0.76 Meters'
     &,8X,'Moderator Mass=',F9.2,' Kg')
      write(1,220)TPM,ENM
```

```
 220    format(1X,'Turbo Pump Mass=',F7.2,' Kg',14X,'Exhaust
       &Nozzle Mass=',F7.2,' Kg')
        write(1,230)RM,ENGMASS
 230    format(1X,'Radiator Mass=',F9.2,' Kg',14X,'Total Engine
       &Mass=',F10.2,' Kg')
        write(1,240)
 240    format(/,1X,'VARIOUS ENGINE PARAMETERS:')
        write(1,250)QR,.06*QR
 250    format(1X,'Reactor Power=',F8.2,' MWatts',11X,'Radiated
       &Power=',F7.2,' MWatts')
        write(1,260)ALPHA,PJET
 260    format(1X,'Engine Specific Mass=',F7.5,' Kg/KWatt',
       &3X,'Jet Power=',F9.2,' MWatts')
        write(1,270)TWR,PJET/QR
 270    format(1X,'Thrust to Weight Ratio=',F7.5,10X,'Engine
       &Efficiency=',F5.4)
        write(1,280)
 280    format(/,1X,'THROAT CONDITIONS:')
        write(1,290)TTH,PTH
 290    format(1X,'Throat temperature=',F8.2,' Kelvin',6X,'Throat
       &pressure=',F7.2,' Atm')
        write(1,295)H2MMT,GAMMAT
 295    format(1X,'Hydrogen Molecular Mass
       &(Throat)=',F5.3,2X,'Gamma (Throat)=',F5.3)
        write(1,300)ATH,VTH
 300    format(1X,'Throat Area=',F7.4,' Sq Meters',11X,'Throat
       &Velocity=',F9.2,' Meters/sec')
        write(1,310)
 310    format(/,1X,'EXIT CONDITIONS:')
        write(1,320)TE,PE
 320    format(1X,'Exit temperature=',F8.2,' Kelvin',8X,'Exit
       &pressure=',f5.3,' Atm')
        write(1,325)H2MME,GAMMAE
 325    format(1X,'Hydrogen Molecular Mass(Exit)=',
       &F5.3,4X,'Gamma(Exit)=',F5.3)
        write(1,330)AE,VE
 330    format(1X,'Exit Area=',F7.4,' Sq Meters',13X,'Exit
       &Velocity=',F9.2,' Meters/sec')
        write(1,340)AMACH
 340    format(1X,'Exit-To-Throat Area Ratio= 300',10X,'Exit Mach
       &Number= ',F5.2)
*************************
* This part of the program uses the method of Masser from Chapter
* 5 to find the constant of integration for the calculation of
* the dose obtained from the plume
*
        f(1)=.07
        f(2)=.00002
        do i=1,2
        BIG=GAMMAE*SQRT(2/(GAMMAE-1)*(2/(GAMMAE+1))**
       &((GAMMAE+1)/(GAMMAE-1)))
        CFCFMAX=(BIG*SQRT(1-(PE/PRES)**((GAMMAE-1)/GAMMAE))+
```

```
      &300*PE/PRES)/BIG
       X=1/(SQRT(PI)*(1-CFCFMAX))
       B=X/4/SQRT(PI)*((GAMMAE-1)/(GAMMAE+1))**.5*(2/(GAMMAE+
      &1))**(1/(GAMMAE-1))
       TOP=f(i)*QR*6.448E+7*(H2MME*PE/TE/82.06)*AMACH*B*
      &AE/PI*10000
       BOTTOM=H2DOT*3.7*(1+(GAMMAE-1)/2*AMACH**2)**.5*(2/
      &(GAMMAE+1))**((GAMMAE+1)/2/(GAMMAE-1))
       CONST=TOP/BOTTOM
350    format(//1X,'lambda=',F15.9,5x,'constant=',f20.4)
       write(1,350)X,CONST
360    format(1X,'f(t)=',f10.7)
       write(1,360)f(i)
       enddo
       close(unit=1)
       stop
       end
```

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NASW-4435

PROJECT WISH: THE EMERALD CITY

PHASE II

June 1991

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ABSTRACT

The purpose of the Permanently Manned Autonomous Space Oasis, designated Project WISH: The Emerald City, is to serve as permanent living quarters for space colonists. In addition, it will serve as a stopover for space missions and will be capable of restationing itself practically anywhere within the solar system to provide support for these missions. The station should be self-sufficient, with no specific dependence on any resources from Earth.

The 1990-1991 design team continued work started by last year's class. This year, further studies were conducted in the areas of orbital mechanics, propulsion, attitude control, and human factors. Critical elements were identified in each of these areas, and guidelines were established for the design of the Emerald City. Using the knowledge gained from these studies, two particular missions of interest, a Saturn Envelope mission and an Earth-to-Mars mission, were examined. The size and mass estimates, along with the methodologies used in their determination, are considered to be the main accomplishments of Phase II.

ACKNOWLEDGEMENTS

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Finally, we would like to mention the students who took part in the design class during the winter quarter. These people were Tim Chu, Doug Gulick, Patrick Plaisted, Keith Rabe, Richard Taylor, and Cathy Trenta.

FOREWORD

This report reflects the result of our design classes through the second year of Project WISH. The design activity was carried out through a sequence of three courses offered during the autumn, winter, and spring quarters. The autumn quarter course was a technical elective, two credit-hour, introductory course, mainly dedicated to the understanding of the accomplishments of the first phase (1989-1990) of the project and building the background to be able to further the design. The winter quarter course fulfilled a four credit-hour design course requirement for the seniors. During the autumn and winter quarters, seminars by NASA Lewis personnel were included. The spring quarter class was a three credit-hour technical elective course which culminated in this report. Project WISH is measuring up to our expectations in every regard both in technical and educational aspects.

Hayrani Oz

Associate Professor

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NOMENCLATURE

Greek

β - thrust angle
γ - true anomaly
Δ - change in
Θ - orientation from inertial reference axes
μ - gravitational parameter, 1.3271544×10^{11} km^3/s^2
π - 3.141592653
ρ - density
σ - stress
τ - period, non-dimensional time
Ω - orbital angular velocity

Latin

D - diameter
F - force
I - impulse, moment of inertia
J - impulse
N - number of crewpeople
P - pressure, power
R - ratio of thrust to gravity force, major radius of torus
TOF - time-of-flight
V - velocity, volume
X - inertial reference axis
Y - inertial reference axis
a - acceleration
g - gravitational constant on surface of Earth, 9.81 m/s^2
h - height, orbital angular momentum
m - mass
m - mass flow rate
n - number of engines, torus spin rate
r - radius
t - time, thickness
x - state vector

Subscripts

RMS - root-mean-square
T - total
atm - atmosphere
c - cavity
con - for control
dry - dry (without propellant)
f - final state
ff - free-flight
i - initial, individual
l - payload

Subscripts (continued)

max - maximum
min - minor
p - propellant
pr1 - first powered flight
pr2 - second powered flight
sp - specific
spe - engine specific
syn - synodic
th - thrust
u - uranium
w - engine
0 - starting state
1 - first state
2 - second state

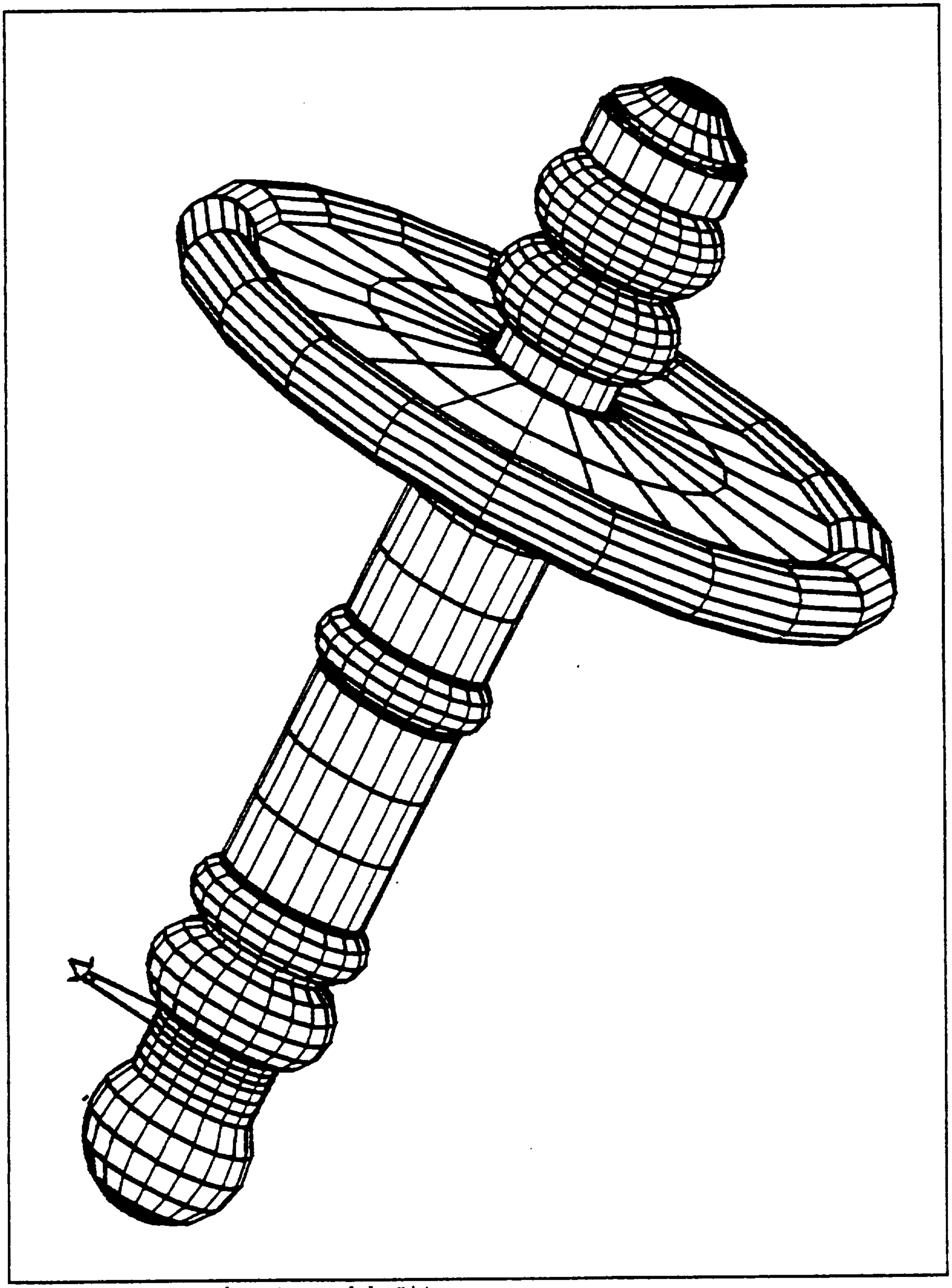

Figure 1.1 - The Emerald City

CHAPTER 1

INTRODUCTION

1.0 PROJECT OVERVIEW

The year is 1620. Winter has struck with a vengeance on the Northeastern coast of the New World. The one hundred and two Pilgrims that have made the journey across the great expanse of the Atlantic Ocean are fighting for their lives. Because there was not time to establish themselves firmly on the shore, the Pilgrims are forced to take shelter on the Mayflower, the ship that brought them across the ocean, for survival. Although some will be lost, those that endure through the first harsh winter will do so because of the support of their ship. With this support, so critical during the initial months of the settlement, the Pilgrims will survive until the spring and establish a permanent colony in the unexplored land of North America.

The year is now 2060. The first manned mission is arriving at the great ringed planet, Saturn. The purpose of this mission is to investigate the Saturnian moon Titan, an object of man's curiosity ever since the Cassini probe first explored it back at the turn of the century. It appears that the speculation about the possibility of colonizing Titan, which started as far back as the 1990's[1], will finally be either proved conclusively or scrapped with the other concepts that never fulfilled their potential. Limitations of the interplanetary ship carrying the expedition have made cargo space precious. Excess amounts of food and supplies have been sacrificed

to accommodate the extensive scientific and data gathering equipment needed to thoroughly explore Titan and the other diverse moons of Saturn. Because of this, the Emerald City has taken up station outside of Saturn's sphere of influence to provide assistance for this historic mission. It will furnish food and equipment for the astronauts, as well as a place for them to take refuge from the desolate vacuum of space. It can also furnish an environment simulating that of Earth, providing artificial gravity as well as the psychological benefits of a large group of people, points which their own ship could not address. Because of the presence of the Emerald City, the mission can remain at Saturn for a much longer period of time. With the extra data that could be collected and recorded, future missions to the moons of Saturn may be made with more confidence, perhaps with the intent of establishing a permanent settlement on Titan.

What is the connection between these two events, separated by over 400 years? The success of both of these scenarios is dependent upon the support provided by the ships in order to make it through the difficult initial stages of their missions. Without this initial support, both missions would be in serious jeopardy. This discussion of past and future events sets the stage for the introduction of Project WISH.

1.1 PROJECT DESCRIPTION

Project WISH (Wandering Interplanetary Space Harbor) is a three year design project currently being conducted at The Ohio

State University. This project deals with the design of a Permanently Manned Autonomous Space Oasis, designated Emerald City (see Figure 1.1). The purpose of Project WISH and the Emerald City is to serve as permanent living quarters for between 500-1000 space inhabitants and to provide crucial support for interplanetary missions in the solar system. To provide this support, the Emerald City will have the ability to relocate itself virtually anywhere in the solar system. This relocation should be accomplished in periods of three years or less whenever possible, thus making the Emerald City flexible in its mission planning. It also should have a lifetime of fifty years, making it an essential link in the space transportation network throughout the latter half of the 21^{st} century.

During the 1990-1991 acedemic year, Phase II of Project WISH was carried out. Phase I, conducted during the 1989-1990 academic year, was a general level study of the major systems required for the Emerald City. During Phase II, a more in-depth study was conducted into the disciplines of orbital mechanics, propulsion, attitude control, and human factors. Critical elements were identified in all of these areas, and guidelines were established for the design of the Emerald City. These guidelines were then combined to carry out the design of two particular missions of interest, a Saturn Envelope mission and an Earth-to-Mars mission. The size and mass estimates of the Emerald City for these missions, along with the methods used to obtain them, are considered the main accomplishments of Phase II, and are the focus of this report.

CHAPTER 2

ORBITAL MECHANICS

2.0 INTRODUCTION

Orbital mechanics is a central discipline in space design, and was one of the areas studied in-depth during Phase II of Project WISH. In general, orbital mechanics deals with the motion of a smaller body, broadly termed as the "satellite", in the gravitational field of a larger body, known as the "central body". In the context of Project WISH, this deals with the motion of the Emerald City in the gravitational field of the sun. Orbital mechanics is used to find the velocity changes, or ΔV's, that are needed for the Emerald City to make a transfer from one place in the solar system to another in a certain time. Last year, some preliminary mission analysis was conducted in an ad hoc manner as an understanding of the computer programs used in the investigation was developed[2]. A preliminary nominal orbit was chosen at 3.2 AU's, but this value was selected due more to the proximity of the asteroid belt than a detailed study of orbital mechanics. Using the experience gained during last year's design effort, a more rigorous study was performed on the transfers for the Emerald City, with the hope that a more substantiated nominal orbit could be chosen. This nominal orbit should yield the most advantageous transfers to as many of the planets as possible. After all, the W in WISH stands for "Wandering", and that is what the Emerald City will be doing throughout the solar system.

2.1 NOMINAL ORBIT

2.1.1 Background

One of the main goals set down for Project WISH this year was the determination of a nominal orbit in heliocentric space. This nominal orbit would act as a sort of "home base" for the Emerald City, a place where it could return between missions for rest and refit. The original design requirement for the project called for a maximum flight time of three years from this chosen orbit to any planet in the solar system. The idea was to make the Emerald City mobile enough to be able to react to orders requiring its presence at a certain planet within a reasonable time frame. However, it was quickly discovered that this requirement placed an untenable burden on the prospective propulsion systems of the Emerald City, even extrapolating well into the 21st century. Thus the design goals were pared down (increasing the flight times to some planets while eliminating trips to other planets altogether) in order to make them more attainable for the time frame being considered.

In the selection of the nominal orbit, an attempt was made to keep the flight times to a minimum while retaining as much of the solar system as possible in the "flight envelope". After much discussion, a new set of design goals emerged for the orbital mechanics and propulsion groups of Project WISH. First of all, it is assumed that the Emerald City will become operational sometime in the mid-21st century. At this time, it is envisioned that humans have become firmly established on the Moon and Mars, and are beginning to look beyond. The next logical step could perhaps be human

exploration of the asteroid belt and the moons of Jupiter. Using such a scenario as background, all of the planets in the solar system were categorized by their perceived importance to humankind's exploration of the solar system in the mid-21^{st} century. These categories were employed as a guide in the decision-making process of what the nominal orbit should be and how much thrust the propulsion system of the Emerald City would need to provide.

The first category is made up of the planets considered very important and consists of Earth, Mars, and Jupiter. These planets would more than likely be the focus of the space exploration effort during the next century. As mentioned earlier, colonies should already be established on the Moon and Mars, and intensive scientific study with preliminary exploration could be taking place on Jupiter and its moons. Thus, the Emerald City's ability to reach them in three years or less would be essential to provide support to these colonies and/or exploration missions. The second category is planets deemed important and is made up of Venus, Saturn, and Uranus. These are planets that would be targets for scientific study, but are not considered as crucial as the planets in the first category. Thus, it was decided that for these planets, transfers of up to five years would be allowable. The third and final category is planets considered to be of least interest, and includes Mercury, Neptune, and Pluto. While these planets have much to offer in terms of scientific value, it was felt that in the time frame of Project WISH, there would be little need for trips to these planets by a spacecraft like the Emerald City. Thus, the

only trips made to these planets would be in special cases when the planets would lie in a favorable orientation with the Emerald City.

By using the above criteria, the design goals given at the beginning of last year were adjusted to more reasonable levels. Instead of requiring the Emerald City to make transfers from the nominal orbit to any planet 100% of the time in three years, this requirement was only applied to the planets in the very important category (Earth, Mars, Jupiter). For the planets in the important category (Venus, Saturn, Uranus), these requirements were eased somewhat by allowing transfers up to five years if necessary, and by not requiring the Emerald City to make the transfer 100% of the time. The design goal would instead be that the Emerald City should be able to make the transfer as much of the time as possible and still keep the propulsion system requirements somewhat reasonable. For the third category (Mercury, Neptune, Pluto), there were no specific design goals set; any transfers that would be possible in less than five years would be considered icing on the cake. Thus, with the above criteria established, the mission analysis was carried out.

2.1.2 Mission Analysis

The main difficulty in the mission analysis portion of Project WISH is the lack of a particular mission to analyze. By definition, the Emerald City is to have the capability to travel wherever it may be needed in the solar system within a reasonable time period. For example, if some crucial product is required for

a fledgling colony on a Jovian moon, the Emerald City will not have the luxury of waiting years for the opening of a launch window to Jupiter before leaving the nominal orbit. Instead, it must be able to make the transfer immediately, regardless of where the planet might be with respect to the Emerald City. This so-called "transfer-on-demand" results in high ΔV's, which is the amount of velocity change that must be provided to the Emerald City in order for it to achieve a transfer from the nominal orbit to a planet. In turn, this high ΔV places a considerable burden on the propulsion system of Project WISH. Thus, it is hoped to keep these mission ΔV's down to reasonable values in order to alleviate some of the propulsion problems associated with large velocity changes (see Chapter 3).

Hence, the question arises as to how to perform the mission analysis for the Emerald City. An ad hoc approach of picking target planets and launch dates for sample cases was discussed, but was ruled out due to its lack of rigor and tedious nature. Instead, a methodology was established that allowed a more thorough investigation of the transfers between a nominal orbit and the planets in the solar system to be performed. This methodology is based on a concept known as the synodic period between two orbiting bodies (see Figure 2.1). The synodic period is the time it takes for a certain angular orientation known as the phase angle between two bodies to repeat itself. For example, as shown in Figure 2.1, at some time $t = 0$, the Emerald City and the target planet will lie in a straight line connecting the two to the sun. At this point, the

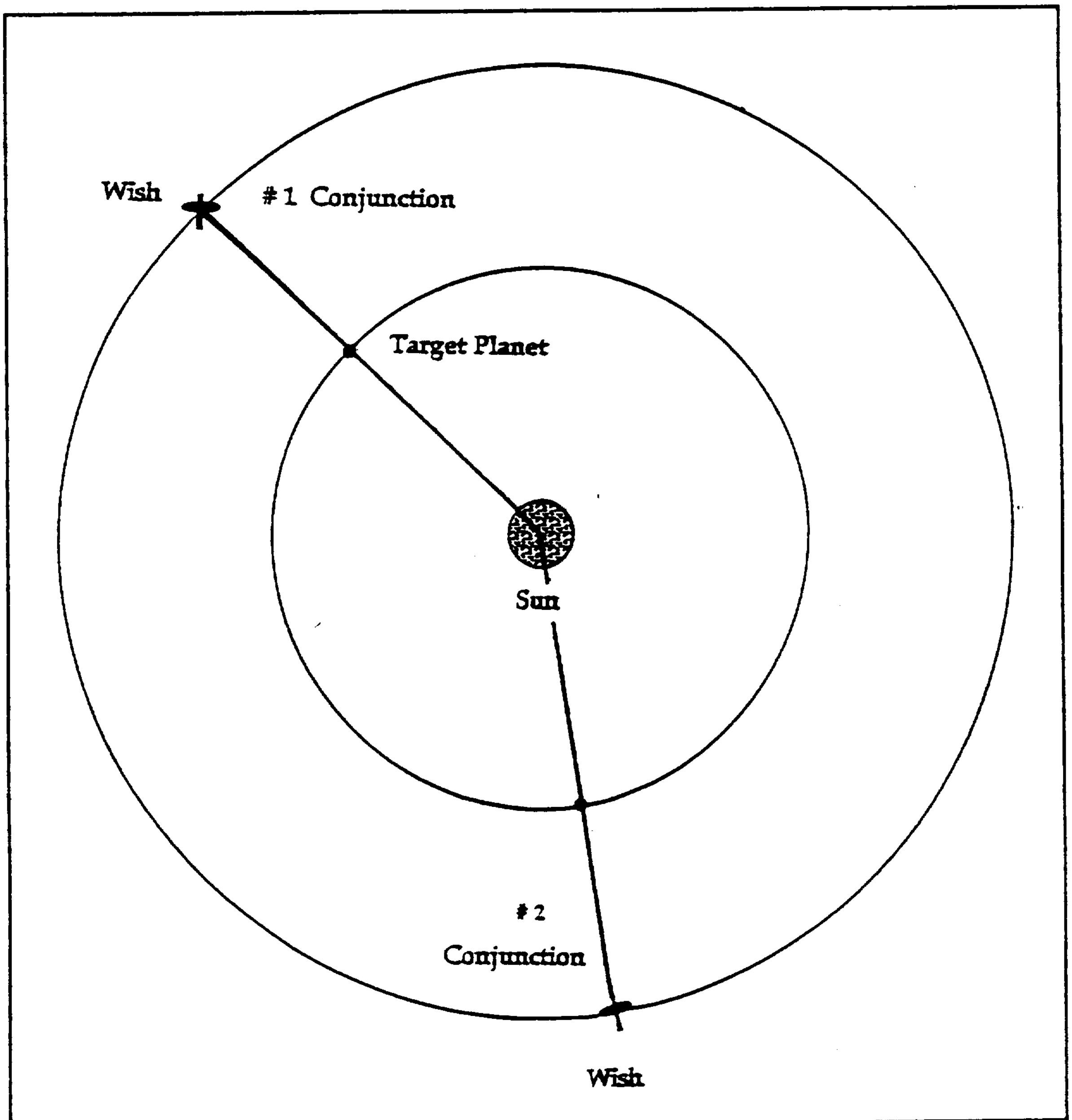

Figure 2.1 - The Synodic Period

phase angle between the two is zero, which is known as conjunction. However, the Emerald City will be travelling slower than the target planet due to the fact that it lies further from the sun. Thus, at a future time, the target planet will essentially "lap" the Emerald City and they will once again be in conjunction at some new posi-

tion in their orbits. The time it takes for this to occur is the called the synodic period, and is denoted by τ_{syn}. The synodic period is related to the angular velocities of the two bodies, Ω_{EC} and Ω_{TO}, and is given by

$$\tau_{syn} = \frac{2\pi}{|\Omega_{TO} - \Omega_{EC}|}. \tag{2.1}$$

Since the synodic period represents the time it takes for a phase angle to repeat itself, it can be extremely helpful in the mission analysis. Because the ΔV required to make a heliocentric transfer between two bodies in a given time is a function of the orientation of the bodies, it will be nearly cyclic in time, with a period that is given by the synodic period. Thus, instead of looking at transfers throughout the entire fifty year time period that the Emerald City is envisioned to be operating in, they can be examined over a single synodic period. This will result in values for ΔV that can be applied to the transfer at any future time. There are two assumptions on the geometry of the orbits that were made in the statement that the ΔV was cyclic with time. The first assumption is that the orbits of the planets are circular instead of elliptical, while the second assumption is that the nominal and planetary orbits all lie in the same plane. If these assumptions were invalid, then the ΔV would not be cyclic with time as was stated, but would depend on where the planet is located in its orbit. It can be seen from Table 2.1[3] that, while these assumptions are not perfect, they are adequate for this preliminary investigation for all of the planets except Mercury and Pluto.

Table 2.1 - Eccentricity and Inclination of the Planets

Planet	Eccentricity	Inclination
Mercury	0.21	7.0^0
Venus	0.01	3.4^0
Earth	0.02	0.0^0
Mars	0.09	1.9^0
Jupiter	0.05	1.3^0
Saturn	0.05	2.5^0
Uranus	0.05	0.8^0
Neptune	0.01	1.8^0
Pluto	0.26	17.1^0

Eccentricity = 0 ==> circular orbit

Inclination = 0^0 ==> same orbital plane as Earth

Using the computer program MULIMP[4] obtained from NASA Lewis Research Center and a simple code written to calculate the synodic periods for a given nominal orbit radius, the mission analysis for Project WISH was conducted. MULIMP is a FORTRAN code that calculates the ΔV required for interplanetary transfers under the assumption that the change in velocity takes place instantaneously, i.e. infinite acceleration. Again, this assumption should not hamper the study because the propulsion system powered-flight times should be sufficiently short with respect to the overall flight time to keep this assumption valid for this precursory investigation. The steps in the mission analysis then proceeded as follows. First, a nominal orbit radius was chosen and input into

the code which calculated the synodic periods for this case. Then, MULIMP was run for each planet at many different launch dates (typically between 100 and 200) over a synodic period and for flight times of one to five years in order to obtain the relationship between the ΔV, the flight time, and the fraction of time into the synodic period that the launch occurred at. This resulted in graphs similar to the one shown in Figure 2.2. This particular graph is for a transfer from a nominal orbit of 3 AU's (astronomical units, 1 AU is distance from Earth to Sun) to Jupiter. It can be seen how the ΔV varies over the synodic period; at some points it is relatively low while at other points, it is quite high. It can also be seen that at the completion of one synodic period, the ΔV is nearly the same as it was at the beginning, as expected. The small difference can be attributed to the two assumptions that were made about the geometries of the orbits. While these assumptions were made to facilitate the analysis, MULIMP uses the actual orbits of the planets, complete with eccentricity and inclination. Thus, the ΔV at each of the endpoints of the synodic period does not match exactly, but the fact that they are so close is a good indication of the validity of the assumptions.

To plot out and analyze graphs such as Figure 2.2 for all of the planets at each nominal orbit considered would be quite tedious. Then, the question would be what would one do with the piles of graphs that had been generated. To aid in the analysis of the data obtained from MULIMP, a program was written that would

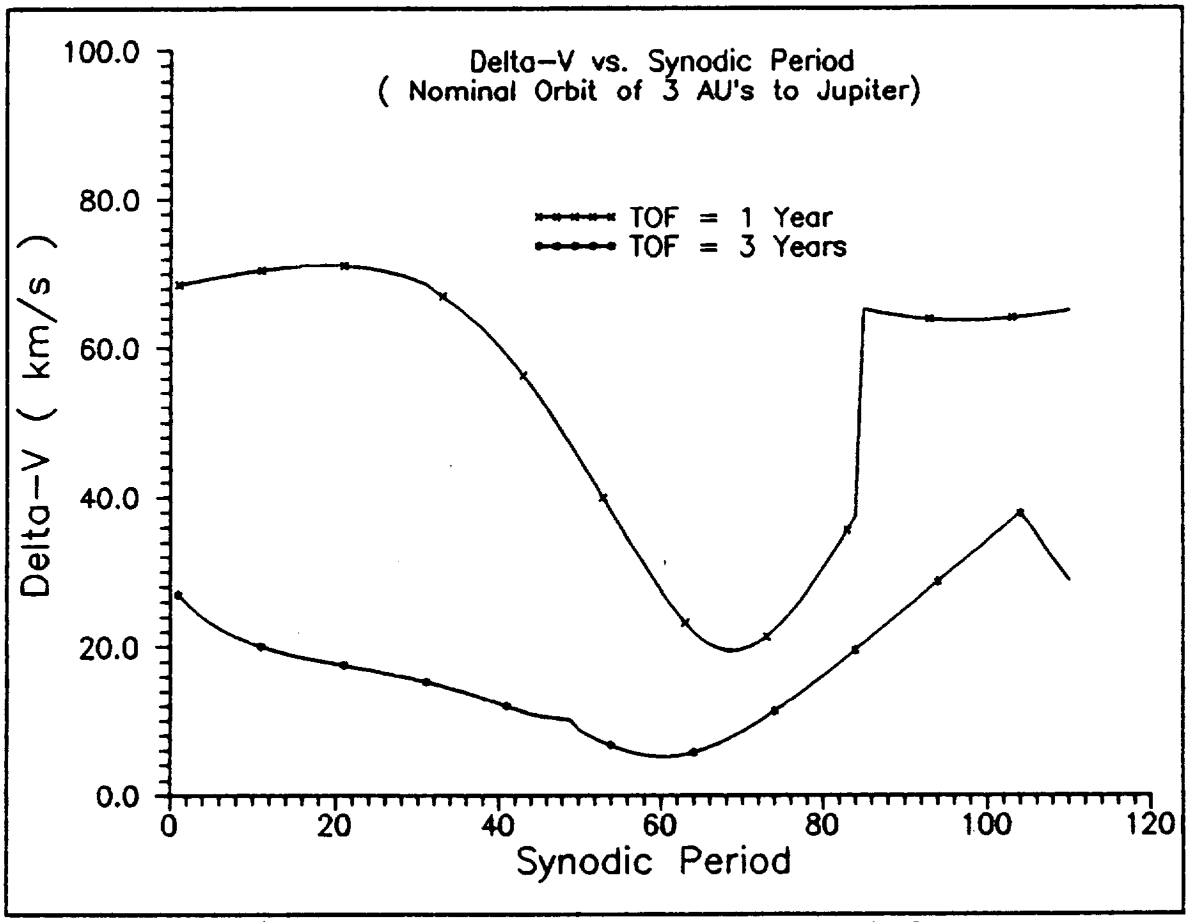

Figure 2.2 - ΔV Behavior Over One Synodic Period (Flight Time is a Parameter)

calculate the percentage of time during the synodic period that a transfer from a nominal orbit to a planet would fall beneath a certain value of ΔV. Then, this data could be presented in the form of a chart, as shown in Table 2.2. The left-hand side of the chart shows what nominal orbit was used, to what planet the transfer was made, and what was the flight time used. For example, in Table 2.2, N3-P5-T3 would mean a transfer from a nominal orbit of 3 AU's to Jupiter (planet #5), with a flight time of three years. The numbers on the right of the chart represent the percentage of time during a synodic period that a transfer falls

Table 2.2 - Percentages When Transfer is Possible Given ΔV Limit
(Middle Seven Planets, Nominal Orbit = 3 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N3-P2-T1	0.0	16.5	38.8	56.5	71.4	85.1	97.6	100
N3-P2-T2	0.0	9.4	33.3	49.8	64.7	78.0	91.4	100
N3-P2-T3	0.0	0.0	26.3	44.3	59.2	73.7	87.5	100
N3-P2-T4	0.0	0.0	20.4	40.0	56.1	70.6	84.3	98.8
N3-P2-T5	0.0	0.0	18.0	36.5	52.9	67.8	82.4	97.6
OPTIMAL FLIGHT TIME = 1.122								
N3-P3-T1	0.0	25.6	50.2	70.0	85.0	98.2	100	100
N3-P3-T2	0.0	28.6	48.0	64.3	78.0	91.6	100	100
N3-P3-T3	0.0	18.1	40.5	57.3	72.7	86.8	100	100
N3-P3-T4	0.0	8.8	34.8	52.9	68.3	83.7	99.1	100
N3-P3-T5	0.0	0.0	31.3	49.3	65.6	81.5	96.9	100
OPTIMAL FLIGHT TIME = 1.394								
N3-P4-T1	0.0	31.5	57.0	85.6	96.3	100	100	100
N3-P4-T2	13.7	44.1	60.7	74.1	87.8	100	100	100
N3-P4-T3	9.3	43.0	61.9	75.6	86.7	98.5	100	100
N3-P4-T4	0.0	28.1	48.9	63.7	78.5	96.7	100	100
N3-P4-T5	0.0	29.3	52.6	68.9	81.5	93.7	100	100
OPTIMAL FLIGHT TIME = 1.764								
N3-P5-T1	0.0	7.3	20.9	29.1	34.5	40.9	92.7	100
N3-P5-T2	11.8	32.7	83.6	99.1	100	100	100	100
N3-P5-T3	24.5	70.0	89.1	100	100	100	100	100
N3-P5-T4	39.1	70.0	87.3	100	100	100	100	100
N3-P5-T5	40.9	68.2	84.5	100	100	100	100	100
OPTIMAL FLIGHT TIME = 4.466								
N3-P6-T1	0.0	0.0	0.0	0.0	0.0	0.0	18.7	29.0
N3-P6-T2	0.0	0.0	8.8	34.7	49.2	100	100	100
N3-P6-T3	0.0	18.1	45.6	94.3	100	100	100	100
N3-P6-T4	0.0	37.8	81.9	100	100	100	100	100
N3-P6-T5	11.9	57.0	87.0	100	100	100	100	100
OPTIMAL FLIGHT TIME = 6.860								
N3-P7-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P7-T2	0.0	0.0	0.0	0.0	0.0	0.0	13.8	33.5
N3-P7-T3	0.0	0.0	0.0	0.0	27.2	48.8	100	100
N3-P7-T4	0.0	0.0	0.0	35.4	75.2	100	100	100
N3-P7-T5	0.0	0.0	29.9	69.7	100	100	100	100
OPTIMAL FLIGHT TIME = 12.960								
N3-P8-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P8-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P8-T3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	13.7
N3-P8-T4	0.0	0.0	0.0	0.0	0.0	18.8	42.6	91.9
N3-P8-T5	0.0	0.0	0.0	0.0	26.9	54.3	100	100

below a certain ΔV. By using charts such as the one found in Table 2.2, different transfer characteristics can be examined quickly and completely, as opposed to a piecemeal approach, which would be time consuming and not as thorough. Thus, now the merits of different nominal orbits can be compared.

2.1.3 Analysis Results

The resulting charts from the mission analysis can be found in Tables 2.3-2.10 for nominal orbits of 3, 4, 5, and 10 AU's. These were broken down into the inner planets in Tables 2.3-2.6 and the outer planets in Tables 2.7-2.10.

Table 2.3 - Percentages When Flight is Possible Given ΔV Limit (Inner Planets, Nominal Orbit = 3 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N3-P1-T1	0.0	0.0	15.1	31.2	45.2	61.3	76.3	90.3
N3-P1-T2	0.0	0.0	8.6	25.8	39.8	54.8	69.9	86.0
N3-P1-T3	0.0	0.0	3.2	21.5	34.4	50.5	65.6	80.6
N3-P1-T4	0.0	0.0	0.0	18.3	32.3	46.2	62.4	78.5
N3-P1-T5	0.0	0.0	0.0	15.1	29.0	44.1	58.1	76.3
N3-P2-T1	0.0	16.5	38.8	56.5	71.4	85.1	97.6	100
N3-P2-T2	0.0	9.4	33.3	49.8	64.7	78.0	91.4	100
N3-P2-T3	0.0	0.0	26.3	44.3	59.2	73.7	87.5	100
N3-P2-T4	0.0	0.0	20.4	40.0	56.1	70.6	84.3	98.8
N3-P2-T5	0.0	0.0	18.0	36.5	52.9	67.8	82.4	97.6
N3-P3-T1	0.0	25.6	50.2	70.0	85.0	98.2	100	100
N3-P3-T2	0.0	28.6	48.0	64.3	78.0	91.6	100	100
N3-P3-T3	0.0	18.1	40.5	57.3	72.7	86.8	100	100
N3-P3-T4	0.0	8.8	34.8	52.9	68.3	83.7	99.1	100
N3-P3-T5	0.0	0.0	31.3	49.3	65.6	81.5	96.9	100
N3-P4-T1	0.0	31.5	57.0	85.6	96.3	100	100	100
N3-P4-T2	13.7	44.1	60.7	74.1	87.8	100	100	100
N3-P4-T3	9.3	43.0	61.9	75.6	86.7	98.5	100	100
N3-P4-T4	0.0	28.1	48.9	63.7	78.5	96.7	100	100
N3-P4-T5	0.0	29.3	52.6	68.9	81.5	93.7	100	100

Table 2.4 - Percentages When Flight is Possible Given ΔV Limit (Inner Planets, Nominal Orbit = 4 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N4-P1-T1	0.0	0.0	0.0	15.2	33.7	51.1	63.0	73.9
N4-P1-T2	0.0	0.0	0.0	27.2	43.5	56.5	66.3	76.1
N4-P1-T3	0.0	0.0	0.0	19.6	39.1	52.2	64.1	73.9
N4-P1-T4	0.0	0.0	0.0	10.9	35.9	48.9	62.0	71.7
N4-P1-T5	0.0	0.0	0.0	8.7	31.5	46.7	59.8	69.6
N4-P2-T1	0.0	0.0	18.3	41.5	61.0	79.3	95.1	100
N4-P2-T2	0.0	12.2	35.4	53.7	68.3	82.9	96.3	100
N4-P2-T3	0.0	7.3	31.7	51.2	64.6	79.3	93.9	100
N4-P2-T4	0.0	0.0	25.6	45.1	59.8	74.4	89.0	100
N4-P2-T5	0.0	0.0	19.5	41.5	58.5	74.4	89.0	100
N4-P3-T1	0.0	0.0	27.1	50.7	75.0	93.6	100	100
N4-P3-T2	0.0	29.3	50.0	66.4	81.4	95.7	100	100
N4-P3-T3	0.0	24.3	45.0	61.4	76.4	91.4	100	100
N4-P3-T4	0.0	17.1	40.0	57.1	73.6	87.9	100	100
N4-P3-T5	0.0	10.7	36.4	55.0	70.7	85.7	100	100
N4-P4-T1	0.0	12.4	34.5	60.2	95.6	100	100	100
N4-P4-T2	10.6	41.6	65.5	85.8	100	100	100	100
N4-P4-T3	7.1	40.7	63.7	77.9	95.6	100	100	100
N4-P4-T4	0.0	31.9	54.0	74.3	94.7	100	100	100
N4-P4-T5	0.0	25.7	54.0	73.5	90.3	99.1	100	100

Table 2.5 - Percentages When Flight is Possible Given ΔV Limit
(Inner Planets, Nominal Orbit = 5 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N5-P1-T1	0.0	0.0	0.0	8.8	26.4	37.4	53.8	69.2
N5-P1-T2	0.0	0.0	7.7	23.1	41.8	54.9	69.2	83.5
N5-P1-T3	0.0	0.0	0.0	23.1	40.7	56.0	68.1	81.3
N5-P1-T4	0.0	0.0	0.0	22.0	38.5	51.6	67.0	79.1
N5-P1-T5	0.0	0.0	0.0	17.6	37.4	50.5	64.8	78.0
N5-P2-T1	0.0	0.0	0.0	21.7	40.8	62.5	83.3	100
N5-P2-T2	0.0	2.5	33.3	51.7	68.3	84.2	100	100
N5-P2-T3	0.0	0.8	31.7	50.0	66.7	81.7	97.5	100
N5-P2-T4	0.0	0.0	26.7	46.7	64.2	79.2	95.8	100
N5-P2-T5	0.0	0.0	24.2	44.2	60.8	77.5	93.3	100
N5-P3-T1	0.0	0.0	0.0	28.9	51.1	78.5	100	100
N5-P3-T2	0.0	23.7	46.7	66.7	83.7	99.3	100	100
N5-P3-T3	0.0	25.9	46.7	65.2	80.7	97.0	100	100
N5-P3-T4	0.0	21.5	43.0	62.2	77.8	94.1	100	100
N5-P3-T5	0.0	17.0	40.0	59.3	76.3	91.9	100	100
N5-P4-T1	0.0	0.0	12.2	31.7	52.5	94.2	100	100
N5-P4-T2	0.0	33.8	64.0	84.2	99.3	100	100	100
N5-P4-T3	9.4	41.7	64.0	83.5	100	100	100	100
N5-P4-T4	0.0	37.4	59.7	77.7	97.1	100	100	100
N5-P4-T5	0.0	34.5	56.1	76.3	97.1	100	100	100

Table 2.6 - Percentages When Flight is Possible Given ΔV Limit (Inner Planets, Nominal Orbit = 10 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N10-P1-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	14.4
N10-P1-T2	0.0	0.0	0.0	4.4	22.2	36.7	51.1	68.9
N10-P1-T3	0.0	0.0	5.6	18.9	36.7	53.3	68.9	87.8
N10-P1-T4	0.0	0.0	11.1	28.9	43.3	58.9	76.7	93.3
N10-P1-T5	0.0	0.0	13.3	30.0	45.6	61.1	77.8	94.4
N10-P2-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	17.2
N10-P2-T2	0.0	0.0	0.0	17.2	35.3	56.0	76.7	97.4
N10-P2-T3	0.0	0.0	15.5	37.9	57.8	75.9	96.6	100
N10-P2-T4	0.0	0.0	27.6	48.3	66.4	84.5	100	100
N10-P2-T5	0.0	0.0	31.9	51.7	68.1	85.3	100	100
N10-P3-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	19.7
N10-P3-T2	0.0	0.0	0.0	22.0	43.3	68.5	95.3	100
N10-P3-T3	0.0	0.0	28.3	50.4	73.2	95.3	100	100
N10-P3-T4	0.0	14.2	39.4	61.4	81.1	100	100	100
N10-P3-T5	0.0	21.3	44.1	64.6	84.3	100	100	100
N10-P4-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21.8
N10-P4-T2	0.0	0.0	0.0	25.9	52.4	87.1	100	100
N10-P4-T3	0.0	0.0	34.7	66.7	94.6	100	100	100
N10-P4-T4	0.0	23.8	53.7	79.6	99.3	100	100	100
N10-P4-T5	0.0	32.0	59.9	83.7	100	100	100	100

Table 2.7 - Percentages When Flight is Possible Given ΔV Limit (Outer Planets, Nominal Orbit = 3 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N3-P5-T1	0.0	7.3	20.9	29.1	34.5	40.9	92.7	100
N3-P5-T2	11.8	32.7	83.6	99.1	100	100	100	100
N3-P5-T3	24.5	70.0	89.1	100	100	100	100	100
N3-P5-T4	39.1	70.0	87.3	100	100	100	100	100
N3-P5-T5	40.9	68.2	84.5	100	100	100	100	100
N3-P6-T1	0.0	0.0	0.0	0.0	0.0	0.0	18.7	29.0
N3-P6-T2	0.0	0.0	8.8	34.7	49.2	100	100	100
N3-P6-T3	0.0	18.1	45.6	94.3	100	100	100	100
N3-P6-T4	0.0	37.8	81.9	100	100	100	100	100
N3-P6-T5	11.9	57.0	87.0	100	100	100	100	100
N3-P7-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P7-T2	0.0	0.0	0.0	0.0	0.0	0.0	13.8	33.5
N3-P7-T3	0.0	0.0	0.0	0.0	27.2	48.8	100	100
N3-P7-T4	0.0	0.0	0.0	35.4	75.2	100	100	100
N3-P7-T5	0.0	0.0	29.9	69.7	100	100	100	100
N3-P8-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P8-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P8-T3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	13.7
N3-P8-T4	0.0	0.0	0.0	0.0	0.0	18.8	42.6	91.9
N3-P8-T5	0.0	0.0	0.0	0.0	26.9	54.3	100	100
N3-P9-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P9-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P9-T3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P9-T4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N3-P9-T5	0.0	0.0	0.0	0.0	0.0	0.0	19.1	40.4

Table 2.8 - Percentages When Flight is Possible Given ΔV Limit
(Outer Planets, Nominal Orbit = 4 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N4-P5-T1	0.0	11.0	18.5	24.0	29.5	36.3	43.8	94.5
N4-P5-T2	12.3	28.1	50.7	99.3	100	100	100	100
N4-P5-T3	22.6	78.8	95.9	100	100	100	100	100
N4-P5-T4	46.6	79.5	94.5	100	100	100	100	100
N4-P5-T5	56.8	80.1	93.8	100	100	100	100	100
N4-P6-T1	0.0	0.0	0.0	0.0	1.4	16.7	20.8	25.0
N4-P6-T2	0.0	0.0	20.1	32.6	41.7	98.6	100	100
N4-P6-T3	0.0	22.2	45.1	99.3	100	100	100	100
N4-P6-T4	2.1	38.2	94.4	99.3	100	100	100	100
N4-P6-T5	18.8	67.4	96.5	100	100	100	100	100
N4-P7-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N4-P7-T2	0.0	0.0	0.0	0.0	0.0	0.0	20.3	29.7
N4-P7-T3	0.0	0.0	0.0	0.0	28.4	42.6	100	100
N4-P7-T4	0.0	0.0	5.4	36.5	70.9	100	100	100
N4-P7-T5	0.0	0.0	31.1	73.0	100	100	100	100
N4-P8-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N4-P8-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N4-P8-T3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	17.1
N4-P8-T4	0.0	0.0	0.0	0.0	0.0	20.2	41.1	94.6
N4-P8-T5	0.0	0.0	0.0	0.0	27.9	49.6	100	100
N4-P9-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N4-P9-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N4-P9-T3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N4-P9-T4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N4-P9-T5	0.0	0.0	0.0	0.0	0.0	0.0	15.4	41.0

Table 2.9 - Percentages When Flight is Possible Given ΔV Limit (Outer Planets, Nominal Orbit = 5 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N5-P5-T1	6.2	15.8	18.1	22.6	24.3	29.4	35.0	40.7
N5-P5-T2	15.8	26.6	40.7	100	100	100	100	100
N5-P5-T3	20.9	57.1	98.9	100	100	100	100	100
N5-P5-T4	32.8	87.6	99.4	100	100	100	100	100
N5-P5-T5	64.4	84.7	98.9	100	100	100	100	100
N5-P6-T1	0.0	0.0	0.0	0.0	12.9	16.3	19.7	22.4
N5-P6-T2	0.0	4.8	22.4	29.3	36.1	97.3	100	100
N5-P6-T3	0.0	23.8	40.1	98.6	100	100	100	100
N5-P6-T4	11.6	36.7	100	100	100	100	100	100
N5-P6-T5	20.4	73.5	99.3	100	100	100	100	100
N5-P7-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N5-P7-T2	0.0	0.0	0.0	0.0	0.0	0.0	20.3	27.2
N5-P7-T3	0.0	0.0	0.0	0.0	27.8	38.0	100	100
N5-P7-T4	0.0	0.0	8.2	35.4	54.4	100	100	100
N5-P7-T5	0.0	0.0	31.6	65.2	100	100	100	100
N5-P8-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N5-P8-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N5-P8-T3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	21.1
N5-P8-T4	0.0	0.0	0.0	0.0	0.0	23.1	38.1	60.5
N5-P8-T5	0.0	0.0	0.0	0.0	29.3	47.6	100	100
N5-P9-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N5-P9-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N5-P9-T3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N5-P9-T4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N5-P9-T5	0.0	0.0	0.0	0.0	0.0	0.0	18.0	39.5

Table 2.10 - Percentages When Flight is Possible Given ΔV Limit
(Outer Planets, Nominal Orbit = 10 AU)

Case ΔV =	10	20	30	40	50	60	70	80
N10-P5-T1	0.0	0.0	0.0	0.0	11.0	14.8	17.4	20.6
N10-P5-T2	0.0	0.0	20.6	27.1	32.9	41.9	100	100
N10-P5-T3	0.0	21.3	36.1	67.1	100	100	100	100
N10-P5-T4	7.7	35.5	94.8	100	100	100	100	100
N10-P5-T5	17.4	54.2	100	100	100	100	100	100
N10-P6-T1	5.2	7.0	7.4	10.0	13.0	13.9	14.3	17.8
N10-P6-T2	6.5	11.3	14.8	19.1	22.2	27.8	33.0	41.7
N10-P6-T3	8.3	20.4	27.0	33.5	44.8	100	100	100
N10-P6-T4	15.7	27.4	35.2	97.0	100	100	100	100
N10-P6-T5	19.1	34.3	97.0	100	100	100	100	100
N10-P7-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N10-P7-T2	0.0	0.0	0.0	0.0	7.8	12.4	15.7	19.0
N10-P7-T3	0.0	0.0	0.0	15.0	19.6	24.8	30.1	36.6
N10-P7-T4	0.0	0.0	16.3	22.9	29.4	38.6	100	100
N10-P7-T5	0.0	10.5	23.5	32.0	87.6	100	100	100
N10-P8-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N10-P8-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N10-P8-T3	0.0	0.0	0.0	0.0	0.0	0.0	15.5	20.8
N10-P8-T4	0.0	0.0	0.0	0.0	15.5	22.6	29.2	36.3
N10-P8-T5	0.0	0.0	0.0	16.7	26.2	34.5	47.6	100
N10-P9-T1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N10-P9-T2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N10-P9-T3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
N10-P9-T4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0
N10-P9-T5	0.0	0.0	0.0	0.0	0.0	0.0	20.9	29.9

Inner Planets

After reading the above discussion and examining Tables 2.3-2.6, one might be tempted to arrive at a conclusion similar to the following. "Well, since planet Earth fell into the first category of planets that had to be reached 100% of the time in three years or less, and according to Tables 2.3-2.6, Earth can be reached 100% of the time in three years for ΔV's greater than 60 km/s, then the propulsion system will have to provide a one-way ΔV of greater than 60 km/s for the design goals to be met." However, upon closer inspection, this conclusion is incorrect due mainly to two reasons. The first reason is that the synodic period of the inner planets are rather low, as can be seen in Table 2.7. They range from about

Table 2.11 - Synodic Periods of the Inner Planets

Planet -	Mercury	Venus	Earth	Mars
τ_{syn} (years)				
n = 3 AU	.25	.70	1.24	2.95
n = 4 AU	.25	.67	1.14	2.46
n = 5 AU	.25	.65	1.10	2.26

one-quarter of a year for Mercury to over two years for Mars. This is a key point in determining what ΔV's will need to be provided for transfers to these inner planets. Because the synodic period is so short, the Emerald City would pass through the entire period in less than three years time. Thus, the Emerald City would have the luxury of waiting at its nominal orbit for some amount of time

after receiving its orders to move, until the ΔV requirements are sufficiently low to make a transfer possible in less than three years and it would still arrive within three years of the time it received its orders to go somewhere.

For example, say that the Emerald City is in a nominal orbit at 4 AU's from the sun. On February 10, 2051, it receives orders to travel to Earth, and its arrival is required within two years. On February 10, the ΔV required to make a two year transfer at this time is found to be 63.9 km/s, a rather high value that would place a large burden on the propulsion system. So instead, the Emerald City decides to wait at the nominal orbit until July 3, 2051, when the ΔV required to make a one year transfer is only 23.1 km/sec. This value is much more easily obtained by the propulsion system, and the Emerald City arrives at Earth on July 2, 2052, about 222 days ahead of the time it was required to be there. This scenario, calculated using MULIMP, is possible because the synodic period for Earth is only 1.14 years, short enough to allow all of the possible phase angles to present themselves to the Emerald City within the time that it is required for it to make the transfer. Thus, instead of leaving immediately, it can wait for some length of time, and then make a transfer that, even with a shorter flight time, has a lower ΔV because of a more favorable orientation.

The second reason that the percentages that a transfer is possible for a given flight time cannot be taken at face value is because of the following fact. In this analysis, the flight time for each transfer is specified, and the ΔV required to make this

exact transfer is found. One might expect that this ΔV would go down as the flight time is made longer, because of the less demanding duration. In general, however, this is not the case for the inner planets. The optimal flight time (the flight time resulting in the lowest ΔV) for the inner planets is on the order of one to two years. Thus, by forcing a trip to one of these planets in, say, three years, the ΔV required actually increases. This is because the trajectory will not go directly to the target planet, but will stall by taking an indirect route in order for the flight time to be the specified value. This phenomenon can be seen in the following example, predicted by MULIMP. The Emerald City receives orders to proceed to Venus on February 27, 2050, and is ordered to be there within 2.5 years. At this point, if the Emerald City were to try to make a transfer with a flight time of 2.5 years, the ΔV required would be 46.1 km/s. However, by making the transfer in only 2.33 years, the ΔV would only be 18.8 km/s, a much better result.

Thus, because of the nature of the transfers to the inner planets for the two reasons cited above, the actual percentages of time that a flight is possible for a given ΔV is rendered meaningless. Instead, what should be looked at is the lowest ΔV a transfer can be made at all with a flight time of three years minus the synodic period. If a transfer can be made at any point in the synodic period for this flight time, then the transfer can be made 100% of the time. Again using the Earth example, in 1.14 years, all of the phase angles between it and the Emerald City will have

occurred due to the length of the synodic period. Thus, if Earth can be reached any percent of the time in 1.86 years or less with a given ΔV, then it can be said that the transfer can be made 100% of the time in under three years from receiving its orders, because it can wait until the favorable orientation appears.

Outer Planets

The results from the mission analysis for the outer planets (Jupiter to Pluto) can be found in Tables 2.7-2.10 for nominal orbits of 3, 4, 5, and 10 AU's. The investigation of the transfers to the outer planets must be treated differently than to the inner planets because the synodic periods are much longer for the outer planets, as seen in Table 2.12. Because of this, the Emerald City

Table 2.12 - Synodic Periods of the Outer Planets

Planet -	Jupiter	Saturn	Uranus	Neptune	Pluto
τ_{syn} (years)					
n = 3 AU	9.26	6.31	5.54	5.37	5.31
n = 4 AU	24.62	10.98	8.85	8.41	8.27
n = 5 AU	196.66	18.02	12.91	12.00	11.71

will encounter only a fraction of the possible phase angles, and must be prepared to make transfers at less than optimum conditions. Thus, for the outer planets, the percentage of time that a transfer is possible for a fixed ΔV becomes important, a yardstick that different nominal orbits can be measured by.

It should be noted what effect moving the nominal orbit further from the sun had on the ability of the Emerald City to reach the outer planets. As seen in Tables 2.7-2.10, there was little benefit gained by moving the nominal orbit away from the sun, and in some cases, the percentages actually went down. This was an important consideration for the selection of the nominal orbit.

2.1.4 Nominal Orbit Selection

The sample nominal orbits that were examined were at 3, 4, 5, and 10 AU's. After examining the percentage tables generated by MULIMP and some discussion, an orbit at 4 AU's was selected as the preliminary nominal orbit. This selection was based on several factors. The first one is the central location of this orbit between Mars, at 1.5 AU and Jupiter, at 5.2 AU. This orbit puts the Emerald City relatively near to the asteroid belt, where vast deposits of valuable materials can be found, and close to the presumed focal points of interplanetary space activity in the mid-21st century, Mars and Jupiter. From this orbit, a three-year transfer to Jupiter can be accomplished 100% of the time with a one-way ΔV of 40 km/s. Thus, if the goals that were laid down at the beginning of this chapter are to be met, the propulsion system would have to provide at least this much ΔV capability. Also, Venus, Earth and Mars can be reached under ΔV's of 40 km/s in three years or less, considering the points that were brought up in the discussion of the inner planets previously. Saturn can be reached

nearly 100% of the time at 40 km/s using flight times of three years, which is an added bonus. To reach Uranus with a ΔV of 40 km/s, flight times of four and five years must be used, and, for five years, the transfer is possible only 73% of the time. This was felt to be acceptable, because Uranus fell into the category of planets deemed marginally important for exploration in the mid-21st century. Mercury can be reached 100% of the time if a trip to that planet is required, but Neptune and Pluto cannot be reached at all by transfers with flight times up to five years and a ΔV limit of 40 km/s. Thus, trips to these outer planets would require special considerations, such as somehow refueling part of the way there or increasing the propulsion capabilities. A nominal orbit of 4 AU's also puts the Emerald City within reasonable distance of Earth for communications and emergencies, which is why a larger radius orbit was not selected (the benefits gained by moving the orbit out were generally negligible).

An interesting point that should be brought up when discussing the necessary ΔV that the propulsion system must provide is the return to the nominal orbit. Early in this study, it was generally assumed that accounting for the return trip would roughly double the maximum ΔV that would be necessary. However, upon further investigation, some doubts were raised as to the validity of this assumption. When using the MULIMP code, the orbit of the Emerald City is entered by using the six classical orbital elements; the eccentricity, the semi-major axis, the inclination, the longitude of the ascending node, the argument of the periapsis, and the time

of periapsis passage. For the circular orbits that were examined, there is no periapsis, and the last element is replaced by some value that locates the Emerald City in the orbit at a given time. When going from the nominal orbit to a target planet, the Emerald City will leave from a particular point on the nominal orbit and arrive at the target planet, and the ΔV can be calculated by MULIMP. However, for the return trip, there is nothing that says the Emerald City has to return to a particular point on the nominal orbit. Instead, it is free to return to any point along the nominal orbit. Unfortunately, when the orbital elements are used to define the nominal orbit, a "ghost point" is created that moves around the defined nominal orbit, as if it were a planet in that orbit. If MULIMP were used to figure out a ΔV required for the return trip, it would calculate it as if it had to return to the "ghost point's" location instead of anywhere on that orbit. Because of this freedom, the Emerald City could return to the nominal orbit at the point that would require the least ΔV for the transfer, and the ΔV's would probably not be as high as originally expected. It was not known if or how MULIMP could be used to calculate the ΔV for a situation like this, so an total ΔV of 50 km/s was assumed as the necessary upper limit to give to the propulsion system. For the inner planets, this figure should be quite adequate due to the Emerald City's ability to make optimal transfers as was discussed above. For the outer planets, it was felt that this figure should result in 100% ability to reach Jupiter and Saturn in three years and above 50% to reach Uranus in

five years (or less, in some cases). Of course, Neptune and Pluto still are unreachable without some special considerations.

2.2 ΔV MINIMIZATION

2.2.1 Introduction

The field of optimization is a powerful tool in the engineering design process. The basic optimization problem is one of determining the minimum or maximum a function, called the objective function, can obtain, subject to certain constraints on its variables. This problem is represented by

$$\min_{x \in R^n} f(x) \tag{2.2}$$

subject to the constraints

$$g_j(x) = 0 \text{ for } j = 1,\ldots.,m_e$$

$$g_j(x) > 0 \text{ for } j = m_e+1,\ldots.,m$$

$$x_l < x < x_u.$$

The function f(x) is the objective function and x is the design variable vector of length n, i.e. n design variables. There are m constraints, the first type being called equality constraints while the second type is called inequality constraints. The third type of constraint is on the design variables themselves and specifies the range of values that these variables can take. There has been much theory developed on techniques to solve optimization problems in the above form. The particular method selected depends upon the nature of the problem; the type of constraints, whether the problem is linear or non-linear, whether the gradients of the functions are to be evaluated analytically or by finite-difference methods, etc..

All of these factors will need to be considered when selecting the computer code to be used to solve the optimization problem.

With the above discussion in mind, an optimization problem studied this year will be discussed, namely the problem of ΔV minimization.

2.2.2 Background

One of the major problems that has surfaced in the first two years of development of Project WISH deals with the transfers from a nominal orbit to the planets in the solar system. It has been noted that the ΔV needed for these transfers have a great impact on the propulsion system requirements. As the values for the ΔV increase, the demands on the propulsion system become considerable. Besides straining the propulsion system, this has a cascade effect on the rest of the systems of the Emerald City; greater propulsion system requirements (thrusts, powered-flight times...) lead to more difficult problems in shielding, structural dynamics, propellant storage, etc... Thus, it is in the best interest of the Emerald City to keep these ΔV's as low as possible. For this reason, it was felt that an optimization of ΔV was justified in order to keep propulsion system requirements as reasonable as possible. It may seem that this would be redundant with the work that was presented earlier in this chapter. However, the analysis presented there used a computer program that assumed an instantaneous change in velocity, with no regard for the powered-flight times, thrust required, or the thrust angles that could be used. By performing

this optimization, it is hoped that these factors can be taken into account, as well as determining the orientation between the Emerald City and the target orbit, typically near the destination planet, that will yield the lowest ΔV.

2.2.3 Assumptions

In order to carry out the optimization, certain assumptions regarding the nature of the orbits and transfers had to be made. The first assumption is that planets are all in circular orbits and are all in the ecliptic plane, i.e. the same plane as the Earth. As was shown in Table 2.1, this is a reasonable assumption for all of the planets except Mercury and Pluto, which are of the least interest in Project WISH. The second assumption made was that there is no loss of propellant during the free-flight phase of the transfer. Of course, there will be some boil-off of the liquid hydrogen due to heat leakage through the storage tanks, but for simplicity, it was ignored for this analysis. The validity of this assumption is questionable, but it was reasoned that the results would still be applicable for this preliminary study. By making this assumption, the acceleration at the end of the first powered-flight phase was set equal to the acceleration at the beginning of the second powered-flight phase.

The third, and most pivotal, assumption is that the trajectory during the powered-flight times is unaffected by the pull of the sun's gravity (see Figure 2.3). During the powered-flight times, the Emerald City is assumed to follow a projectile-like course,

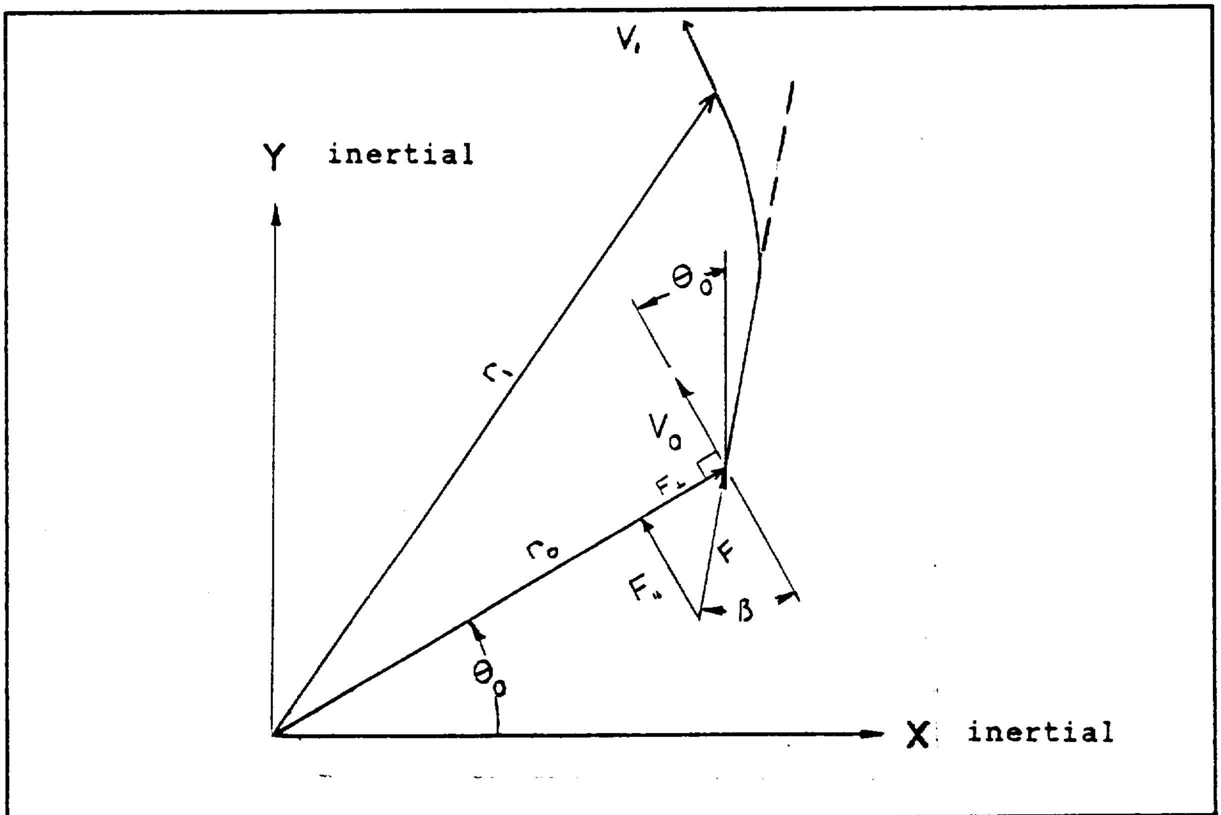

Figure 2.3 - Assumption for Powered-Flight Phases

determined by the X- and Y-components of the initial velocity and acceleration (thrust) vectors. This assumption is based on the value of the ratio of accelerations due to thrust and gravity during the powered-flight phases, given by

$$R(t) = \frac{a_{th}}{a_g} = \frac{F_{th} r(t)^2}{\mu m(t)} \qquad (2.3)$$

$$m(t) = m_o - \dot{m}t \quad \mu = 1.327 \times 10^{11} \frac{km^3}{s^2}.$$

It can be seen that R is dependent upon the thrust, F_{th}, the radius, r(t), and ship mass, m(t). For representative values for the Emerald City, R will be a very large number, and thus it was felt that this assumption was valid for this preliminary investigation.

2.2.4 Geometry

The geometry that was used for the ΔV optimization is shown in Figure 2.4. The technique used to model the interplanetary transfer was to break down the transfer into three arcs. The first and last arcs represents the initial and terminal powered-flight phases, respectively, while the middle arc represents the free-flight phase. In the powered-flight portions, the thrust level, powered-flight times, and thrust angles govern the trajectory, while in the free-flight portion, the classical relationships governing orbiting bodies are used (more on these relationships can be found in Appendix A). The variables shown in Figure 2.4 are defined in Table 2.13.

Table 2.13 - Definition of Variables Used in ΔV Minimization Problem

r_0	- nominal orbit radius
r_1	- radius at end of first powered flight, beginning of free-flight
r_2	- radius at end of free-flight, beginning of second powered flight
r_f	- target orbit radius
θ_0	- angle between inertial reference frame X-axis and r_0 vector
θ_1	- angle between inertial reference frame X-axis and r_1 vector
θ_2	- angle between inertial reference frame X-axis and r_2 vector
θ_f	- angle between inertial reference frame X-axis and r_f vector
θ_i	- angle between inertial reference frame X-axis and target planet at launch
γ_0	- angle between inertial reference frame X-axis and eccentricity vector (line of apsides)
γ_1	- true anomaly of free-flight orbit at point 1
γ_2	- true anomaly of free-flight orbit at point 2

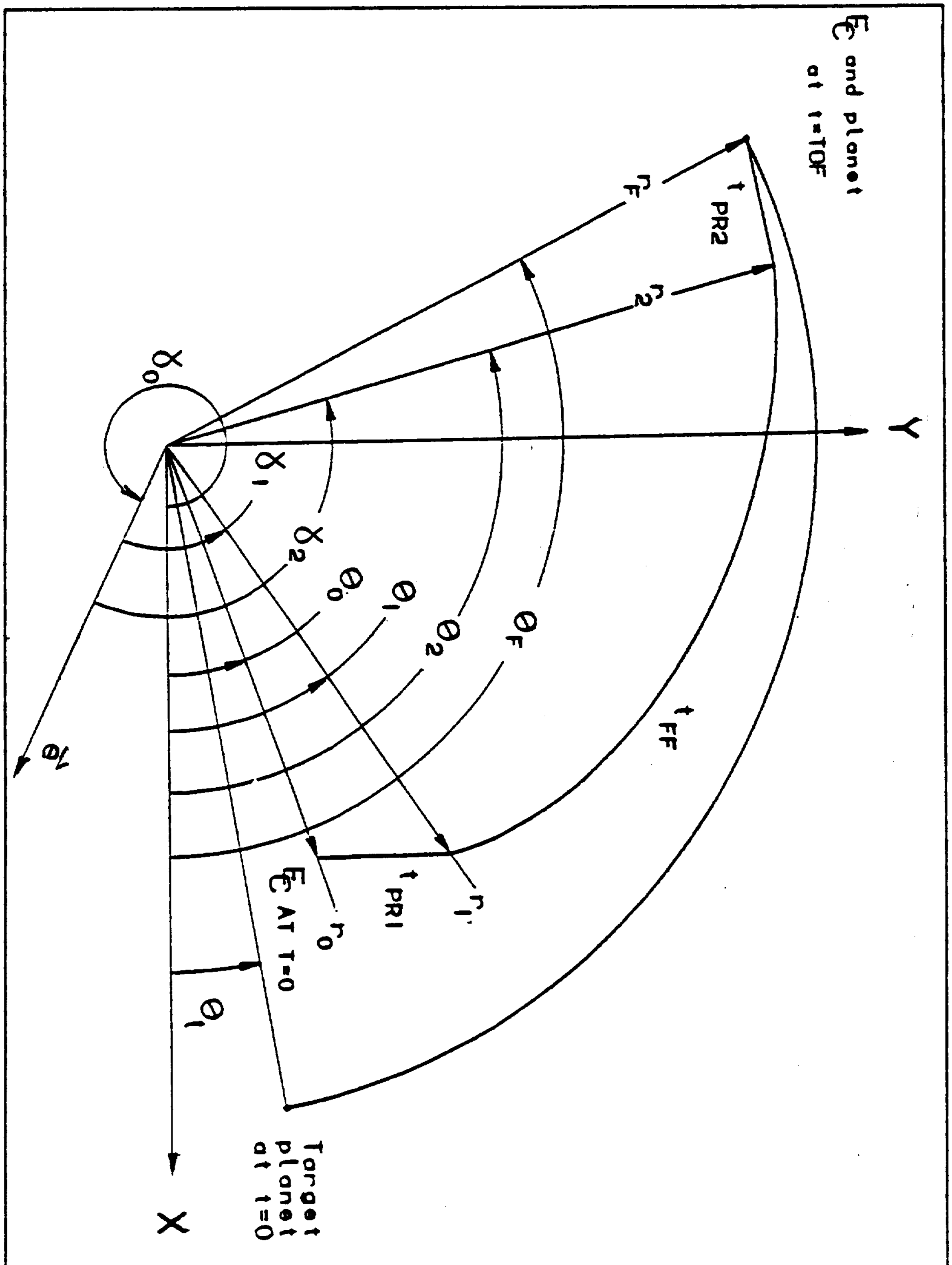

Figure 2.4 - Geometry for Optimization Problem

2.2.5 Methodology

In an optimization problem, there is a objective function that represents the variable to be optimized and constraint functions that represent the restrictions on the problem. For the ΔV minimization problem, the general objective function is

$$\Delta V_T = \Delta V_1 + \Delta V_2 = (\vec{V}_1 - \vec{V}_o) + (\vec{V}_f - \vec{V}_2) \qquad (2.4)$$

where V_1, V_0, V_f, and V_2 are the velocities at points 1, 0, f, and 2, respectively (see Figure 2.4). The general constraints for this problem are given by constraints on the total time-of-flight, the acceleration levels, the powered-flight time lengths, the rendezvous requirements, and physical variable constraints. The acceleration and powered-flight-time constraints arise from the limitations of the propulsion system, while the total time-of-flight constraint governs the allowable time to make the transfer. The rendezvous constraint requires that the position and velocity of the Emerald City at the end of the transfer matches the position and velocity of the target orbit near the destination planet at that time, and the final constraint is to ensure that there are no physically meaningless solutions found, such as negative times or negative magnitudes. Thus the problem is to find the equations that relate v_0, v_1, v_2, and v_f to the design variables chosen and determine equations for the constraints listed above.

The first step that was taken was to define a state vector for point i, x_i, such that

$$\vec{X}_i = \{\vec{r}_i \quad \vec{v}_i\}. \qquad (2.5)$$

The values at the end of each powered-flight phase can be related to the values at the beginning of each powered-flight phase by

$$\vec{x}_1 = \vec{x}_o + \Delta\vec{x}_1$$
$$\vec{x}_f = \vec{x}_2 + \Delta\vec{x}_2 \qquad (2.6)$$

where

$$\Delta\vec{x}_j = \{\Delta\vec{r}_j \quad \Delta\vec{v}_j\}. \qquad (2.7)$$

By examining the equations governing the trajectory of the Emerald City during the powered and free-flight phases, and by contemplating the nature of the problem, eight design variables were identified. These variables are the two powered-flight times, t_{pr1} and t_{pr2}, the two thrust angles, β_1 and β_2, the acceleration level, a_0, the true anomaly at the end of free-flight, γ_2, and the initial orientations of the Emerald City and the target planet, θ_0 and θ_i, respectively. The functional dependencies between these variables and the state vectors were found to be

$$\vec{x}_1 = \vec{x}_o(\theta_o) + \Delta\vec{x}_1\ (\theta_o, a_o, t_{pr1})$$
$$\vec{x}_f = \vec{x}_2(\gamma_2)|_{\vec{x}_1} + \Delta\vec{x}_2(\gamma_2, a_o, t_{pr1}, t_{pr2}) \qquad (2.8)$$

where the vertical bar signifies dependance on a previous state. After applying equations for ΔV_1 and ΔV_2 as found in Appendix A, the objective function was written as

$$\Delta V_T = -I_{sp}g[\ln(1 - \frac{a_o t_{pr1}}{I_{sp}g}) + \ln(1 - \frac{a_o t_{pr2}}{I_{sp}g - a_o t_{pr1}})], \qquad (2.9)$$

where I_{sp} is the specific impulse of the propulsion system (see Chapter 3), and g is the gravity constant on the surface of the Earth, 9.81 m/s^2. The constraint functions may then be written as

1. $t_{ff} + t_{pr1} + t_{pr2} < TOF_{max}$
2. $a_0 < a_{0,max}$
3. $t_{pr1} + t_{pr2} < t_{pr,max}$
4. $x_{f,EC} = x_{f,TO}$.
5. $a_0,\ t_{pr1},\ t_{pr2},\ t_{ff} > 0$.

It turns out that the objective function is dependant only on three of the eight design variables, namely a_0, t_{pr1}, and t_{pr2}. The other five design variables enter into the problem through the first and fourth constraints above. These constraints on the total time-of-flight and the rendezvous requirement incorporate the rest of the design variables through the equations governing the powered and free-flight phases of the trajectory. These equations can be found in detail in Appendix A.

Once the objective function and constraint functions are determined, an optimization code can be utilized to solve this problem. A potential optimization subroutine was identified, but due to time and personnel constraints, the problem was not programmed. This subroutine solves a general, non-linear optimization problem using a successive quadratic programming algorithm and finite difference gradients. It is hoped that this optimization problem can be programmed during next year's design effort and some meaningful results obtained.

2.3 CONCLUSIONS

This year, a detailed study was performed on the orbital mechanics of Project WISH. Based on this study, a nominal orbit of 4 AU's was selected, which places the Emerald City roughly 600,000,000 kilometers from the sun, between the orbits of Mars and Jupiter. From this orbit, it was determined that the planets Mercury, Venus, Earth, Mars, Jupiter, and Saturn can be reached 100% of the time in flight times of three years or less with a ΔV limit of 40 km/s one-way. Uranus can be reached 73% of the time with this ΔV in flight times of five years, which was felt to be adequate for this project. Neptune and Pluto cannot be reached at all at this ΔV with flight times of up to five years, but it was believed that there would be little need for a ship such as the Emerald City to make trips to these planets during the time frame considered. The ΔV minimization problem was also examined and equations were generated that yield the trajectory of the Emerald City during both the powered- and free-flight phases. The objective and constraint functions were determined, but due to a lack of time, the problem was not programmed into an optimization code. It is hoped that in the future, this problem can be solved and meaningful results can be obtained. Also, more work should be done on determining the ΔV's required to return to the nominal orbit from a planet. With additional study in this area, a more accurate round-trip figure for the ΔV's can hopefully be obtained. As of the end of this study, a total round-trip ΔV of 50 km/s was used as the upper ceiling for the Emerald City.

CHAPTER 3

PROPULSION

3.0 INTRODUCTION

The propulsion system of the Emerald City plays an important role in the success of Project WISH. For this reason, it was one of the most rigorously studied subsystems this year. At the conclusion of the Phase I design conducted last year, an antimatter engine was selected as the main propulsion system of the Emerald City[2]. However, it was decided during Phase II that antimatter was too theoretical for the time frame of Project WISH, and that the propulsion system chosen should be more readily procurable by the mid-21st century. With the hope of bringing the choice of the propulsion system into the Emerald City's time frame, the propulsion team looked at several areas of interest in spacecraft propulsion. First, a general level study was conducted in order to determine some of the important parameters associated with spacecraft propulsion and what impact they had on the choice of a particular system. Once this was accomplished, a propulsion system was chosen and a specific propulsion study was conducted in order to determine the significant characteristics of this system. From this study, it was possible to calculate other important parameters that were dependent upon the choice of a particular propulsion system. Finally, this propulsion system was used in the design of two representative missions, which may be found in Chapter 6.

3.1 GENERAL PROPULSION STUDY

3.1.1 Theoretical Background

The purpose of this general level study is to determine which parameters play a major role in the choice of a propulsion system, and what the orders of magnitude of these parameters need to be for an application such as Project WISH. The study is independent of the type of engine that will be used for Project WISH; only the physical nature of spacecraft propulsion enters into this investigation. The first step in the analysis was to determine the effects that the mission requirements have on propulsion parameters. Since the Emerald City is expected to be a large spacecraft, the propulsion system will be required to produce high impulse levels, J, at high specific impulses, I_{sp}. High specific impulse, which is the impulse produced per unit weight of propellant expelled, is desirable because it corresponds to less propellant required to accomplish a mission. The I_{sp} plays an important role in the equation for the propellant mass ratio, given by

$$\frac{m_p}{m_o} = 1 - e^{-\frac{\Delta V}{I_{sp} g}} \qquad (3.1)$$

which is plotted in Figure 3.1. This figure demonstrates how the propellant mass ratio, which is the fraction of the total initial ship mass consisting of propellant, decreases as the I_{sp} is increased or the ΔV is decreased. Thus, to keep the propellant mass required for a given mission down, a propulsion system capable of generating a high I_{sp} should be utilized.

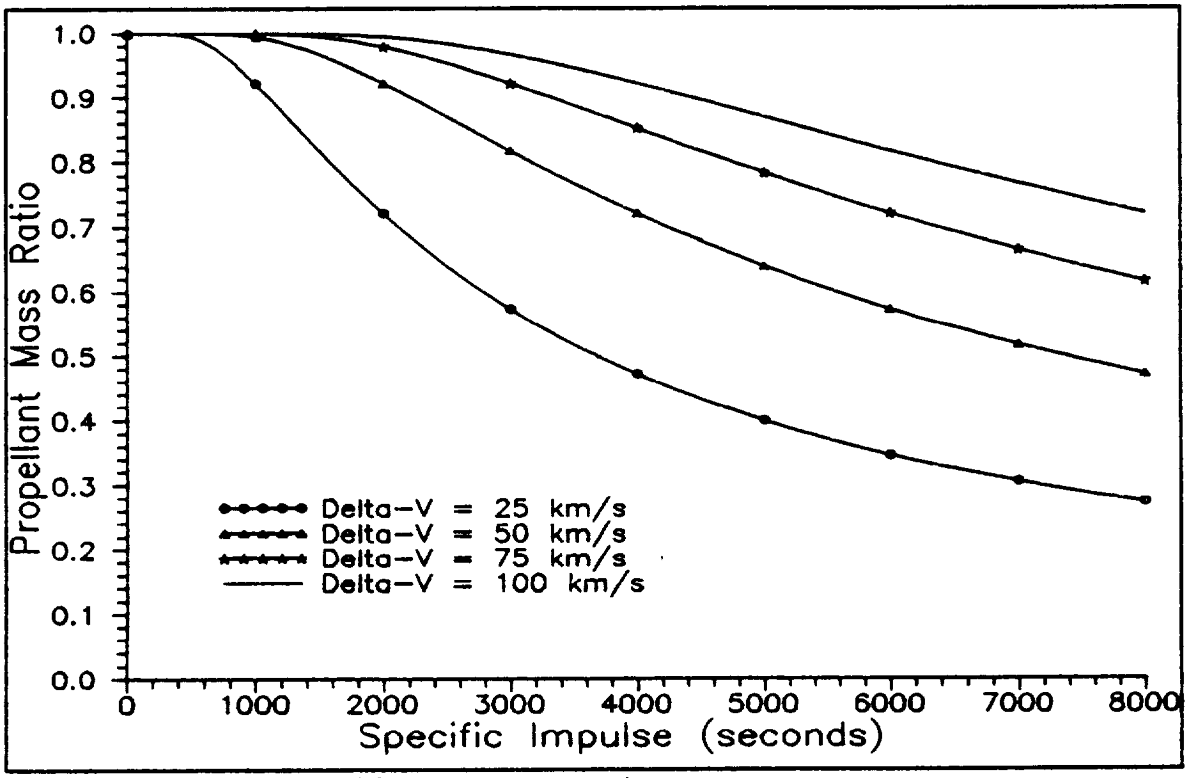

Figure 3.1 - Propellant Mass Ratio vs. I_{sp} With ΔV as a Parameter

However, for the Emerald City, a high-I_{sp} engine alone is not enough. An engine capable of producing high impulse levels as well as the high I_{sp}'s is needed. The impulse required for a mission is a function of the dry mass of the ship (no propellant included) and the ΔV and is given by

$$J = m_{dry}\, I_{sp}\, g\, (e^{\frac{\Delta V}{I_{sp}\, g}} - 1) \qquad (3.2)$$

which is plotted in Figure 3.2. From this figure, it can be seen that the impulse required for an Emerald City mission will be very large, on the order of 10^{13} to 10^{14} newtons. This is due to two factors; the large dry mass of the Emerald City, and the high ΔV levels that will be required for its missions (see Chapter 2). The

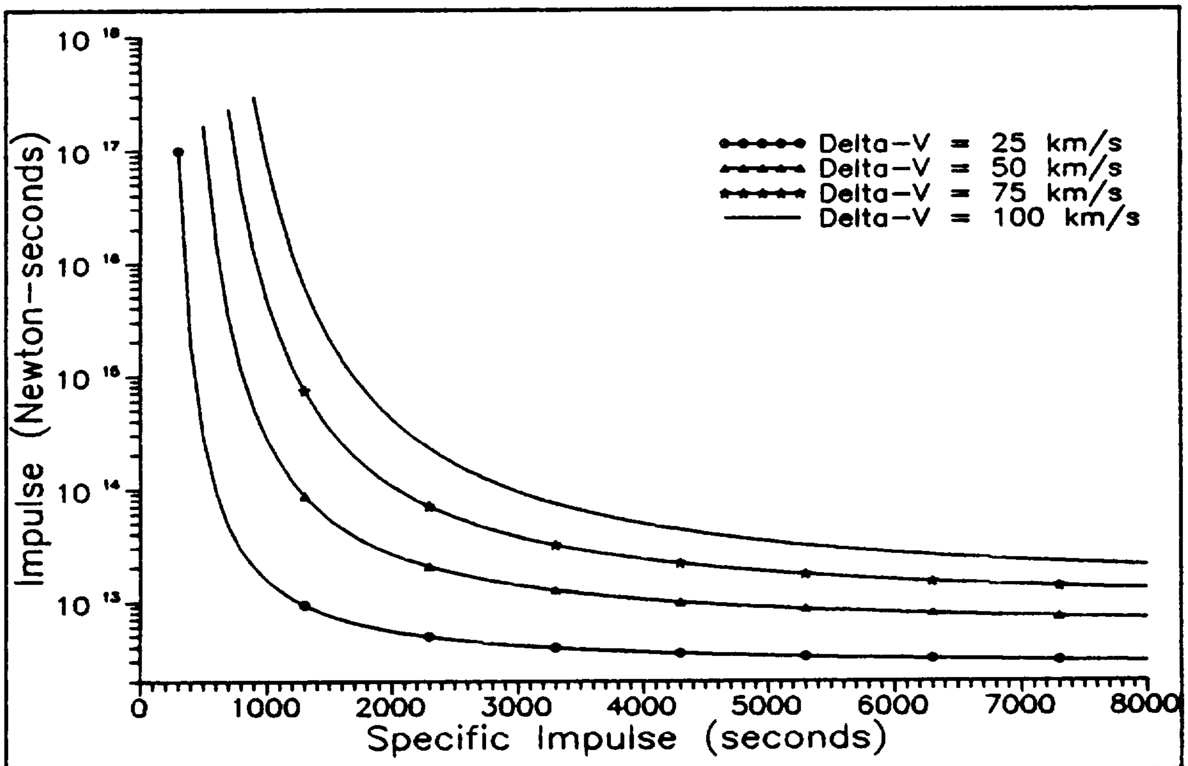

Figure 3.2 - Impulse vs. Specific Impulse with ΔV as a Parameter and $m_{dry} = 10^8$ kg

effect that the I_{sp} has on the impulse required can also be seen from Figure 3.2. As the I_{sp} goes down, the impulse required goes up, eventually approaching infinity as I_{sp} approaches zero. This is because of the extra propellant mass that must be carried at the lower I_{sp}'s, which makes the Emerald City heavier and more difficult to move. Since impulse is the product of a force and a time, high impulses would necessitate high thrusts, F, and high powered-flight times, t_{pr}. Neither of these are desirable from an engine maintenance/lifetime viewpoint. It may also be seen from Figure 3.2 that, if the Emerald City were to burn for its entire flight time (~ 3 years or ~ 10^8 seconds), the thrust that would be required would still be on the order of 10^5 newtons.

Once it has been determined that high I_{sp}'s and high thrusts are needed, the next step was to search for a propulsion system that could satisfy these requirements. A general level study was conducted in an attempt to find such a system from choices that included chemical, electrical, nuclear, and antimatter. However, antimatter had already been ruled out for being too theoretical in nature, even for an advanced project such as Project WISH. Thus, the remaining systems were analyzed in terms of the specific impulse and thrust levels that each system could provided.

3.1.2 General Propulsion Systems

Chemical propulsion was the first type of propulsion analyzed and was promptly rejected. Although the thrust values were adequate, the specific impulses of these systems fell far short of the requirements that the Emerald City would place on them. Electrical propulsion was the next system that was studied. These systems produced high enough I_{sp}'s, but were severely limited in the amount of thrust that could be produced. Nuclear thermal propulsion (NTP) was the final type of system examined. Estimates of the nuclear propulsion systems of the 21st century gave the best combination of high thrust and high I_{sp} that would be required for Project WISH. Thus, NTP was examined in more detail as a primary propulsion system for the Emerald City.

One of the main points considered when examining nuclear systems was the question of fission or fusion. The propulsion group decided that fusion, like antimatter, would be too

theoretical for the Emerald City. Another drawback of using fusion propulsion is the mass penalty that would accompany the system. Compared to the fission systems, the fusion systems are estimated to be substantially more massive. Of course, a breakthrough in fusion research could lead to the development of an operational fusion system suitable for the Emerald City, but for now, fusion was rejected as a main propulsion candidate. Fission systems have the advantage of being more advanced and currently less massive than their fusion counterparts. Unfortunately, associated with fission is a rather troublesome radiation problem, which leads to an increase in the shielding mass required for human safety. Also, the more developed types of fission systems have relatively low I_{sp}'s for this project, and the more advanced engines with the higher I_{sp}'s are still conceptual in nature. With all of these considerations in mind, it was decided that a fission nuclear propulsion system would be used for the Phase II design of the Emerald City.

Within the fission category of NTP, there are two major subgroups; solid-core reactors and gas-core reactors. As its name suggests, a solid-core NTP system utilizes a fissioning solid-core of uranium to generate thermal energy. This energy is transferred to a lightweight propellant, usually hydrogen, which is then expelled through a convergent-divergent nozzle. However, the core is limited in temperature by material considerations, and thus the achievable I_{sp} is similarly limited. Current estimates and some testing during the NERVA (Nuclear Energy Rocket Vehicle Appli-

cation) place the solid-core I_{sp} in the range of 800-1100 seconds[5]. The gas-core NTP system attempts to overcome this issue of limited temperature by allowing the core temperature to increase beyond the boiling point of uranium, thus resulting in a gaseous core. This increase in temperature enables more thermal energy to be transmitted to the propellant, which results in a higher possible I_{sp}. Current theoretical studies give a possible I_{sp} for this conceptual engine on the order of 2000-8000 seconds[6]. It was felt that the I_{sp} of the solid-core rockets were simply too low for the needs of Project WISH (see Figures 3.1 and 3.2) and thus the gas-core NTP system, with its higher estimated attainable thrusts and I_{sp}'s was chosen for further study as the main propulsion system of the Emerald City.

There were two major types of gas-core nuclear rockets (GCNR) studied, the light bulb (closed-cycle, LBGCNR) and space radiator (open-cycle, SRGCNR). The closed-cycle gas-core engine has its gaseous core encased in a transparent material that allows radiation from the fissioning core to heat the propellant while keeping the propellant and core physically apart. The open-cycle gas-core engine has the propellant and core in direct contact, which allows for better heat transfer, but also leads to a loss of uranium from the core. In determining which of these propulsion systems was best for the Emerald City, three main engine parameters were investigated, the I_{sp}, the thrust per engine, F_i, and the mass per engine, m_{Wi}. Table 3.1 shows some comparison figures of these three parameters for the two systems considered[6]. This table shows that

the closed-cycle engine has a significantly lower specific impulse than the open-cycle engine. However, the maximum expected thrust value for the closed-cycle system is higher than that of the open-cycle engine. It will be shown later in this chapter in Figure 3.5 that the advantage of producing higher thrust decreases as the thrust level increases. It was felt that the closed-cycle gas-core engines still had I_{sp} values that were too low for use by the Emerald City, and thus, the open-cycle engine (hereafter referred to as SRGCNR) was chosen as the propulsion system for the Emerald City. The values of thrust and I_{sp} that were used as representative figures for the SRGCNR in the remainder of this report are[7]

Table 3.1 - Comparison of I_{sp}, F, and m_e for LBGCNR and SRGCNR

LBGCNR DATA

Thrust (N)	m_{vi} (kg)	I_{sp} (sec)
133,370	14,050	1780
1,334,200	34,475	2355
4,002,800	385,500	2635

SRGCNR DATA

Thrust (N)	m_{vi} (kg)	I_{sp} (sec)
22,240	36,280	2400
177,900	101,440	5500
444,750	213,350	6000

$$I_{sp} = 5000 \text{ seconds}$$

$$F_i = 4.4 \times 10^5 \text{ N.}$$

These values fell within the range of thrusts and I_{sp}'s that were discussed in Reference 7, and were felt to be reasonably obtainable if the SRCGNR is ever developed. Once the propulsion system and important parameters were determined, it was then possible to conduct a detailed study of the SRCGNR system of the Emerald City.

3.2 SPACE RADIATED GAS-CORE NUCLEAR ROCKET STUDY

With the specific propulsion system determined and values for I_{sp} and F_i known, it was then possible to perform a more detailed study of this system. Because the SRCGNR is so conceptual in nature, there are many unanswered questions about its form and function. Figure 3.3 is a schematic of the GCNR that was conceptually developed at NASA Lewis Research Center by Mr. Robert Ragsdale[7]. The main mechanical components of this engine are the pressure shell, the moderator, the turbopump, the nozzle, and the

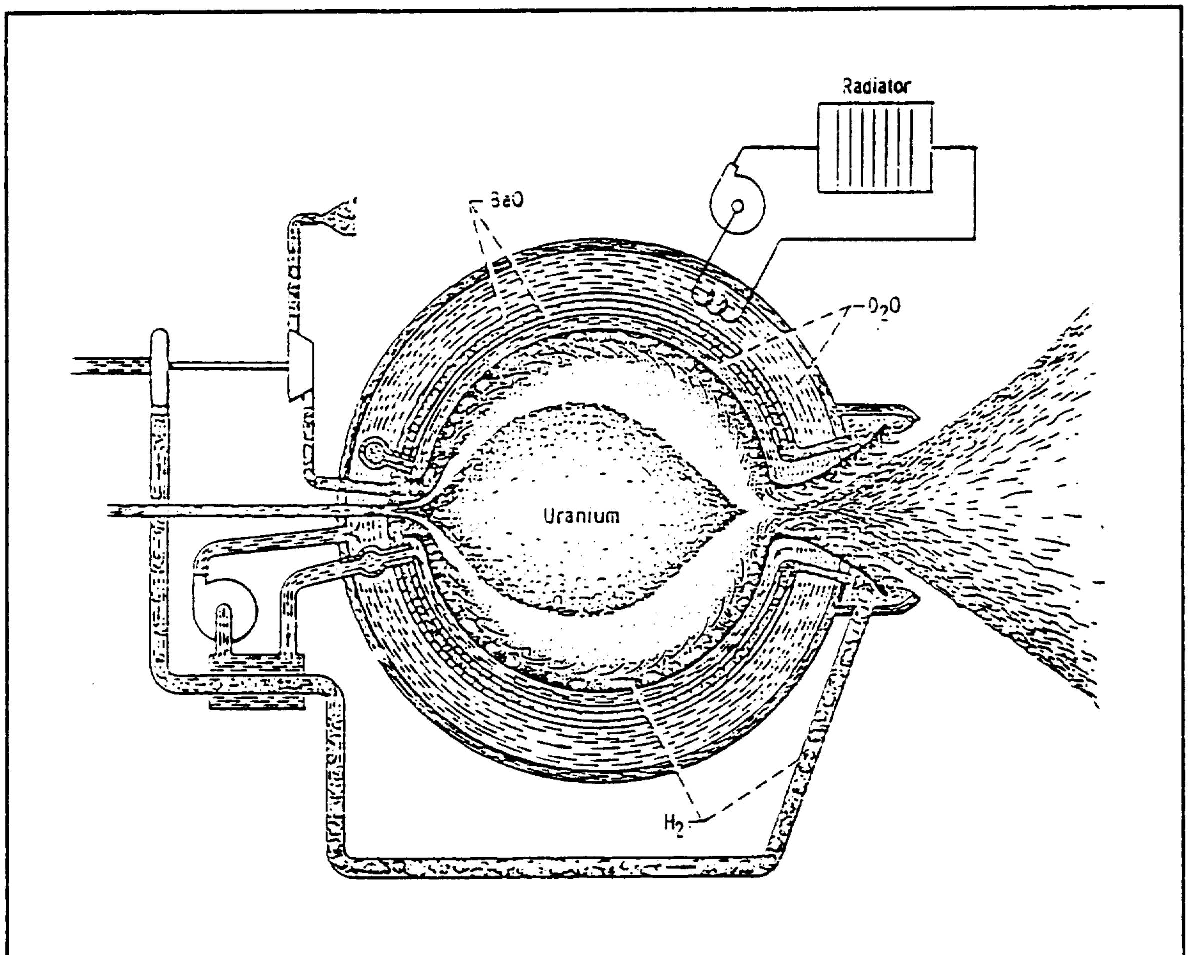

Figure 3.3 - Conceptual Sketch of the Gas-Core Nuclear Rocket

radiator, which work together in the following manner. First, liquid hydrogen is pumped through the moderator by the turbopump to aid in heat removal there. Then, the hydrogen is pumped into the cavity where the uranium isotope, U^{233}, is undergoing the fission process. The hydrogen absorbs heat from the fissioning uranium and is expelled through the nozzle to produce thrust. The moderator is used to reflect and reduce the energy of (thermalize) the neutrons which aids in sustaining the fission process. The pressure shell encloses the core, cavity, and moderator, while the radiator is used to remove any excess heat that builds up in the engine which cannot be removed by the hydrogen alone.

Once a working knowledge of the engine had been acquired, it was then possible to compute various important engine characteristics. These computations were done using a computer program written during this study, which can be found listed in Appendix B. This program was used to generate the important parameters of the SRCGNR from general equations and equations given by Mr. Ragsdale for estimating the mass of an engine[7]. The results obtained from this program may be found tabulated in Table 3.2. The program has four inputs that need to be determined before the other parameters may be calculated. These inputs are I_{sp}, F_i, cavity diameter, D_c, and the ratio of the volume of the gaseous uranium core to the volume of the cavity, V_u/V_c. For the data in Table 3.2, these values were selected to be I_{sp} = 5000 seconds, F_i = 4.4 x 10^5 newtons, D_c = 3.3 meters, and V_u/V_c = 0.23. These figures were considered to be reasonable estimates of the operating conditions

Table 3.2 - Engine Parameters of a SRGCNR, F_i = 4.4 x 10^5 N, I_{sp} = 5000 s, D_c = 3.3 m, and V_u/V_c = 0.23

SPECIFIED VALUES:

I_{sp}=5000.00 s   Thrust=440000.00 N V_u/V_c=.23 D_c=3.30 m

CALCULATED ENGINE PARAMETERS:

ENGINE CAVITY CONDITIONS:

T(H_2)=25165.49 K	T(U^{233})=76183.45 K
Cavity Pressure=981.25 atm	Critical Mass U^{233}=40.12 Kg
Cavity Volume=18.82 m^3	Uranium Volume=4.33 m^3
Uranium Diameter=2.02 m	Uranium Density=9.27 Kg/m^3
Hydrogen Molecular Mass=0.759	Gamma=1.261

MASS FLOW RATES:

H_2-To-U^{233} Ratio=393.89	Propellant=8.97 kg/s
H_2=8.951 kg/s	U^{233}=0.023 kg/s

ENGINE DIMENSIONS:

Pressure Shell Thickness=0.087 m	
Pressure Shell Mass=52597 kg	
Moderator Thickness=0.76 m	Moderator Mass=46632 kg
Turbo Pump Mass=773 kg	Exhaust Nozzle Mass=233 kg
Radiator Mass=138463 kg	Total Engine Mass=238682 kg

VARIOUS ENGINE PARAMETERS:

Reactor Power=16185.08 MW	Radiated Power=971.10 MW
Specific Mass=0.02224 kg/KW	Jet Power=10787.32 MW
Thrust to Weight Ratio=0.188	Engine Efficiency=0.6665

THROAT CONDITIONS:

Temperature=22258.87 K	Pressure=524.48 atm
Hydrogen Molecular Mass=0.843	Gamma=1.246
Area=0.0022 m^2	Velocity= 16542.23 m/s

EXIT CONDITIONS:

Temperature=4120.51 K	Pressure=0.115 atm
Hydrogen Molecular Mass=2.004	Gamma=1.226
Area=0.6489 m^2	Velocity= 49033.25 m/s
Exit-To-Throat Area Ratio=300	Mach Number= 6.50

of the SRGCNR engine[7]. With these inputs, the program was able to calculate the conditions in the cavity, in the throat, and at the exit of the engine. The formulas that were used to calculate these conditions are included with the program in Appendix B. Some of the more interesting results noted from Table 3.2 are a throat temperature of 22,260 K, a jet power of 10.8 gigawatts, a reactor power of 16.1 gigawatts, a thrust-to-weight ratio of 0.19, and an exit velocity of 49 km/s. This value of exit velocity is over ten times what a standard chemical rocket can attain today. The overall mass of the engine was found to be 238.7 metric tonnes.

Once the SRCGNR characteristics and values were obtained, it was then possible to proceed with other calculations that were related to the propulsion system. For example, the total number of engines, n, that would be required for any given mission had to be determined. This formula, given by

$$n = \frac{m_{dry}\, I_{sp}\, g}{F_i\, t_{pr}} \left(e^{\frac{\Delta V}{I_{sp} g}} - 1\right) \tag{3.3}$$

was found by replacing J in Equation 3.2 with $nF_i t_{pr}$. Once the number of engines was determined, it was possible to calculate the total engine mass from

$$m_w = n\, m_{wi}. \tag{3.4}$$

Equation 3.3 is plotted in Figure 3.4, which shows the relationship between the number of engines required and the mission ΔV for a range of different powered-flight times. This figure was generated using an I_{sp} of 5000 seconds, an F_i of 4.4×10^5 newtons,

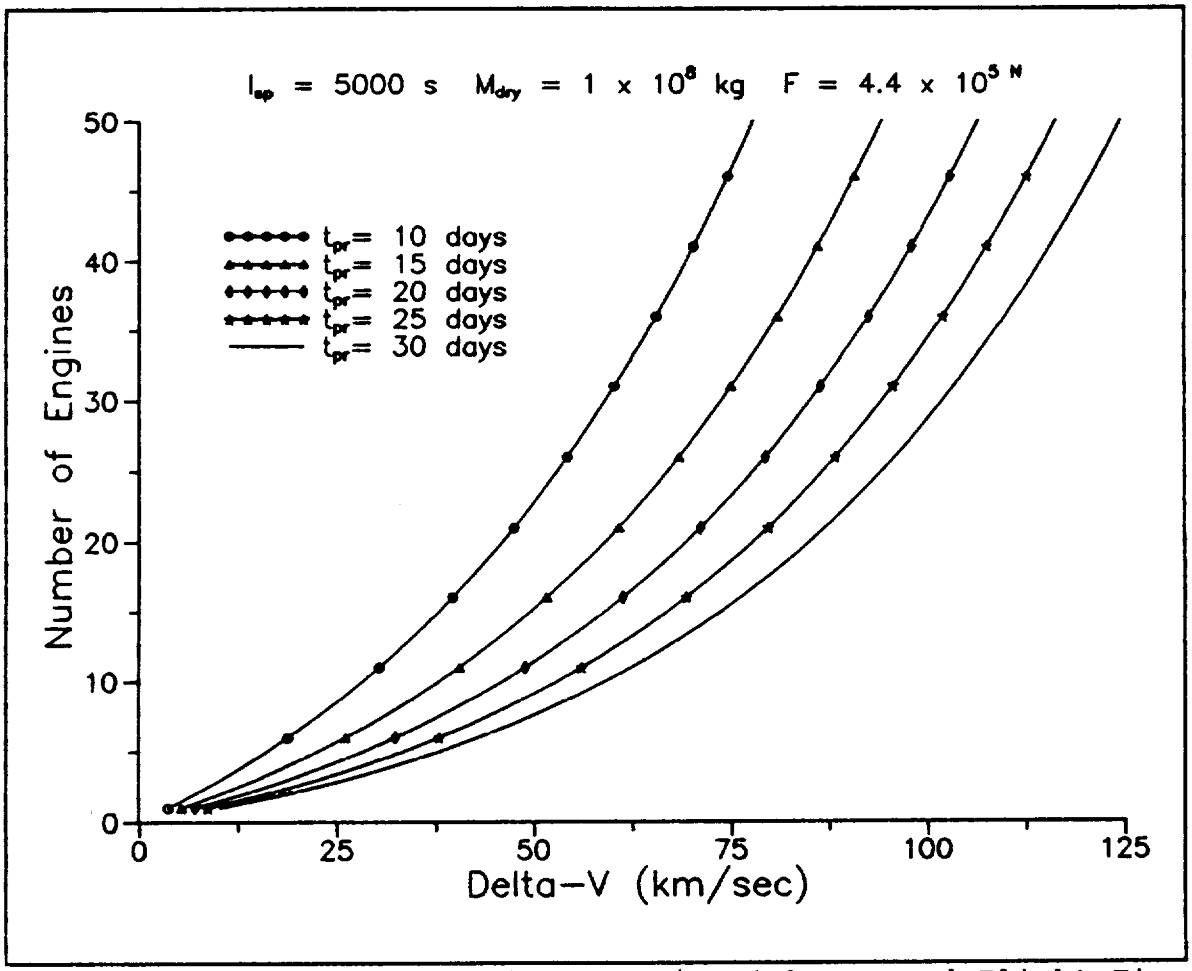

Figure 3.4 - Number of Engines vs. ΔV with Powered-Flight Time a Parameter

and an assumed dry mass of 1 x 10^8 kg. By using Equation 3.3, the number of engines for any selected mission parameters (ΔV, t_{pr}, m_{dry}) may be determined. Once the total engine mass has been found, it was then possible to calculate the payload mass in the form of the payload mass ratio, given by

$$\frac{m_l}{m_o} = e^{-\frac{\Delta V}{I_{sp}g}} - \frac{I_{sp}g}{I_{spe}} \left(1 - e^{-\frac{\Delta V}{I_{sp}g}} \right). \qquad (3.5)$$

In Equation 3.5, I_{spe} is defined as the engine specific impulse and is defined as the impulse produced per unit mass of the engine, or

$$I_{spe} = \frac{F_i \, t_{pr}}{m_{wi}} \tag{3.6}$$

Equation 3.5 was used to generate Figure 3.5, which graphically shows the relationship between the payload mass ratio and the engine specific impulse I_{spe} for different mission ΔV's. An interesting note about this plot is that the payload mass ratio

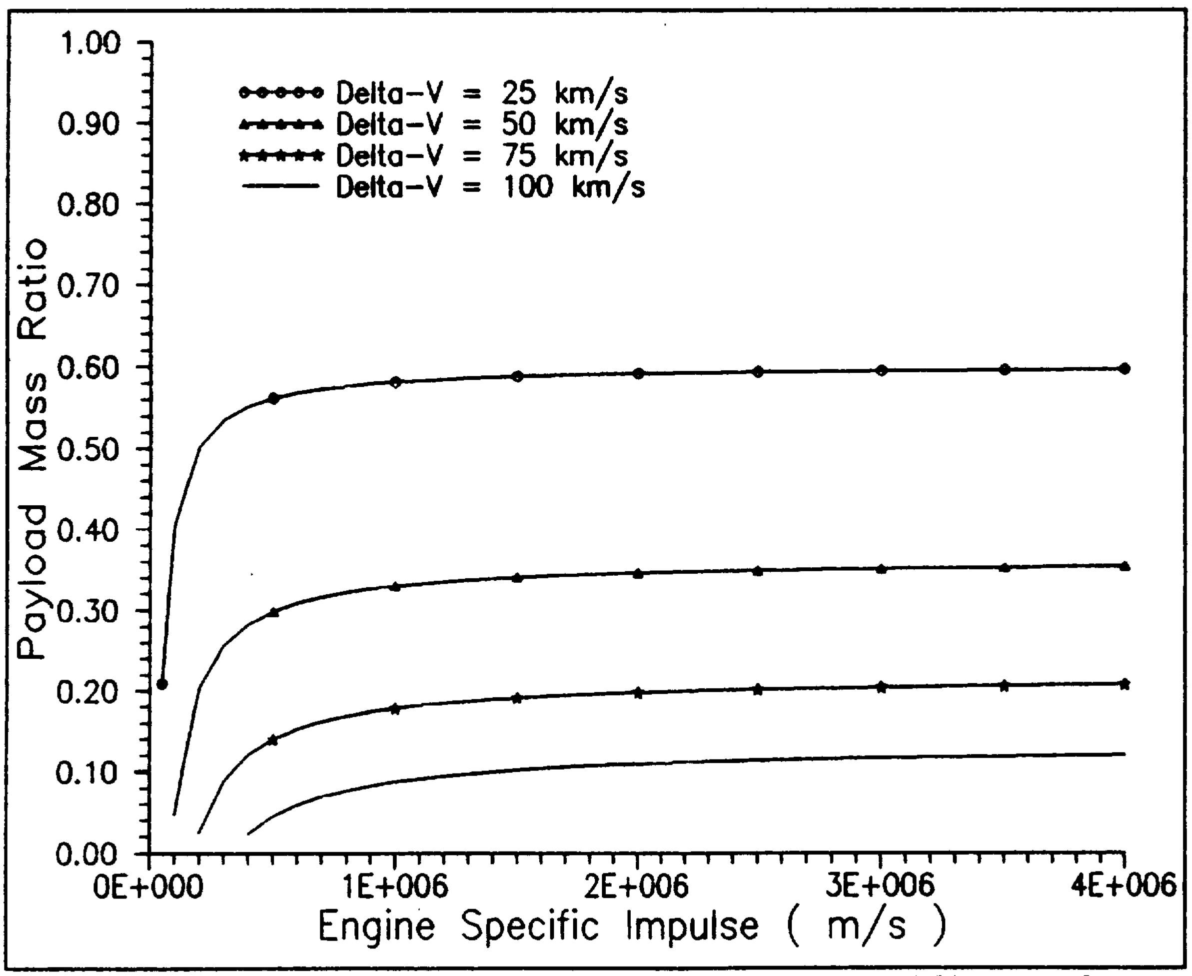

Figure 3.5 - Payload Mass Ratio vs. Engine Specific Impulse (I_{spe}) with ΔV as a Parameter.

tends to level out no matter how much the engine specific impulse is increased. Thus, an increase in the I_{spe} is only beneficial when operating in the region where the payload mass ratio substantially increases. There is little to be gained by increasing the thrust or the powered-flight time or decreasing the engine mass once the I_{spe} has reached the point where the payload mass ratio begins to level off. Also, it may be noted from Equation 3.5 that there is a value of I_{spe} below which, the engine mass exceeds the entire payload mass, and thus the payload mass ratio will be negative.

3.3 CONCLUSIONS

The end result of the analysis presented in this chapter was to find equations that would yield the total mass of the propulsion system and propellant required. By using the equations and figures given here, these masses can be found for different mission specifications. This is a crucial step in the design of the Emerald City for specific missions. Two missions were designed for the Emerald City, using the equations derived here. These missions can be found in Chapter 6.

The preliminary propulsion system that was chosen was the space radiated open-cycle gas-core nuclear rocket engine. The benefits of this system are its high I_{sp} and thrust values that can be obtained. However, this engine is still very conceptual in nature, and so it is not currently known if the idea will eventually lead to a functioning rocket engine or not. It can be seen from Table 3.2 that the temperature in the throat of the

nozzle is predicted to be around 22,000 K!. Clearly, advances will have to be made in the development of materials capable of withstanding this kind of thermal load. Another area in need of development is the confinement of the gaseous core. Currently, it is envisioned that the core will be confined by a vortex of inert gas. This area will need to be thoroughly researched if the concept of an open-cycle gas-core reactor is ever to reach fruition.

There were two detrimental side effects noted by using the open-cycle engine, both of which are related. The first of these effects was the loss of uranium out of the cavity and through the nozzle. It was estimated that approximately 2 metric tonnes of uranium will be lost per engine per day of powered-flight time. The second side effect is the environmental and human factors concern of the radiation generated by this type of engine. Large amounts of radiative material are present in the exhaust plume which will result in massive shielding requirements for the crew. Both of these problems can be lessened by going to a light-bulb gas-core engine, which is a closed-cycle type of engine. However, this engine is not foreseen to have an I_{sp} as high as that of the open cycle engine. The light bulb engine could prove useful if the scope of Project WISH were to ever be reduced, i.e. fewer people, less massive ship, smaller ΔV's... Until then, however, the open cycle gas-core engine will remain as the only type of nuclear engine capable of generating the high impulse-high I_{sp} combination needed for Project WISH in the mid-21st century.

CHAPTER 4

ATTITUDE CONTROL

4.0 INTRODUCTION

Attitude control is an important facet of Project WISH due to the fact that it will be necessary to damp out oscillations in attitude due to disturbances. Because the Emerald City will be spinning to produce artificial gravity (see Chapter 5), the system will be gyroscopic in nature. Therefore, stable equilibrium configurations had to be determined, which was done during the Phase I study conducted last year. Once these configurations were known, attitude control studies were conducted in order to find the control requirements needed to return the Emerald City to equilibrium following a disturbance. The methodologies used to perform these attitude control studies are the focus of this chapter.

4.1 ATTITUDE DYNAMICS BACKGROUND

For a body that is as large and complex as the Emerald City, there will be some structural deformations that arise due to the loads encountered. However, for the purpose of this preliminary investigation, the Emerald City was considered to be a rigid body, i.e. there are no deformations present. Although structural dynamics was not considered in this analysis, it was examined last year using a lumped-mass modelling method, and will again be considered in detail during the third year (1991-1992) of the

project. Since the goal of the attitude control study was to obtain an order of magnitude estimation of the control thruster requirements, it was felt that the rigid body assumption was adequate.

As stated earlier, the stable equilibrium configurations for the Emerald City were determined from last year's vehicle dynamics study[2]. These stable configurations were found to be dependent on the parameters b and r in the following manner:

1. $b > 1/r$ and $b > 4/r - 3$
2. $b < 1/r$ and $r > 1$
3. $b < 4/r - 3$ and $r < 1$.

The parameters b and r are given by

$$r = I_z / I_x = \text{slenderness ratio}$$

$$b = n / \Omega = \text{spin rate / orbital angular rate}$$

where I_z and I_x are the mass moments of inertia about the z- and x-axes (see schematic in Chapter 6), and n is the spin rate of the torus. Thus, it may be seen that the stability of the Emerald City depends upon three principal factors: mass moments of inertia (determined by ship configuration), spin rate (determined from artificial gravity considerations), and orbital angular velocity (determined by the nominal orbit).

4.2 CONTROL SYSTEM REQUIREMENTS

Once the stable configurations of the Emerald City are known, the control aspect of the problem may be examined. The main goal

of the attitude control study was to determine the values of thrust, root-mean-square power, and propellant mass required to damp out an initial disturbance.

4.2.1 State Feedback Control Design

The first step in achieving this goal was to determine the state-space equations that model the motion of the Emerald City. This equation was expressed in the non-dimensional state-space formulation[8]

$$\dot{\hat{x}} = [A][\hat{x}] + [B][\hat{T}] \tag{4.1}$$

where

$$[A] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ (1-r) & 0 & 0 & (2-r) \\ 0 & (1-r) & (r-2) & 0 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{4.2}$$

$$[X] = [\hat{\theta}_1 \;\; \hat{\theta}_2 \;\; \dot{\hat{\theta}}_1 \;\; \dot{\hat{\theta}}_2]$$

and $[\hat{T}]$ is the non-dimensional torque provided by the control thrusters (^ denotes non-dimensionalized quantities). Equation 4.1 is used to simulate the state response of the Emerald City for given ship configurations, thrust input configurations, and initial disturbances. A linear quadratic regulator control program (MATLAB[9] software) was used to obtain the non-dimensional state

response that minimizes the control design performance index, defined by

$$CDPI = \frac{1}{2}\int_0^\infty (\hat{X}^T\hat{X} + \hat{T}^T\hat{T})\, d\tau . \tag{4.3}$$

By minimizing this performance index, the program minimizes the total control effort and the kinetic energy of the system and yields a linear state feedback control law

$$[\hat{T}] = [G][\hat{x}] \tag{4.4}$$

where G is the control feedback gain matrix minimizing the CDPI in Equation 4.3.

4.2.2 Attitude Control Power Required

For the control torques of Equation 4.4, the non-dimensional control power consumed can be found by the non-dimensional power S^*,

$$S^* = \int_0^\infty \hat{T}^T \vartheta^{-T} \vartheta^{-1}\hat{T} d\tau \tag{4.5}$$

which yields

$$S^* = \hat{X}_o^T P^* \hat{X}_o \tag{4.6}$$

for any initial disturbance state x_0. The power matrix P^* is the solution of the linear matrix equation

$$A_{cl}^T P^* + P^* A_{cl} + G\, \vartheta^{-T} \vartheta^{-1} G = 0 \tag{4.7}$$

where

$$A_{cl} = A + BG \tag{4.8}$$

and the non-dimensional thruster distribution matrix is defined by

$$\hat{\boldsymbol{\theta}} = D_t^{-1}[D] \tag{4.9}$$

in which D_t is the torus diameter and the elements of [D] are the moment arms of respective input forces yielding moments about the centroidal body axes x and y of the vehicle which are parallel to the torus plane. The required quantities and matricies in Equations 4.4 through 4.9 can be evaluated by using any suitable control software such as the MATLAB Control Toolbox mentioned previously.

The various dimensional and non-dimensional quantities are related by

$$[\,\theta_1 \;\; \theta_2 \;\; \dot{\theta}_1 \;\; \dot{\theta}_2\,] = [\,\hat{\theta}_1 \;\; \hat{\theta}_2 \;\; \hat{\dot{\theta}}_1 \;\; \hat{\dot{\theta}}_2\,]\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & n & 0 \\ 0 & 0 & 0 & n \end{bmatrix} \tag{4.10}$$

$$t_{con} = \frac{\tau_c}{n} \tag{4.11}$$

$$F_i = \hat{F}_i \frac{I_z\, n^2}{D_t}. \tag{4.12}$$

$$T = \hat{T}\, I_z\, n^2. \tag{4.13}$$

The non-dimensional control thrust forces can be obtained via

$$\hat{F}_{con} = \hat{D}^{-1}\hat{T} \tag{4.14}$$

In all of the above equations, a ˆ above a variable denotes non-dimensional quantities. The root-mean-square power required to damp out a disturbance can then be found from the value of S^t, computed via Equations 4.6 and 4.7. The root-mean-square control power required is then given by the formula

$$P_{rms} = \frac{v_{ex} I_s n^2}{2D_t} \sqrt{\frac{S^*}{\tau_c}} \tag{4.15}$$

where v_{ex} is the exit velocity of the control thrusters.

4.2.3 Attitude Control Propellant Requirement

The propellant mass required for the control thrusters can be found by the formula

$$m_{p,con} = \left(\frac{I_s n}{D_t v_{ex}}\right) \sum_{j=1}^{f} \int_0^{\tau_c} |\hat{F}_j| \, d\tau \tag{4.16}$$

where f is the total number of inputs used. The integral in Equation 4.16 is found by numerically integrating the non-dimensional thrust input versus non-dimensional time plots (see for example Figure 6.4 for specific mission designs)

4.2.4 Attitude Control Thruster Configuration

For a body with the size of the Emerald City, one would need hundreds, perhaps thousands, of control thrusters to exert attitude control torques about the body axes. The number would depend on

the thrust output capabilities of the control thrusters. Taking into account so many thrusters individually in the attitude control problem study would result in a very large D (thruster distribution) matrix. However, because the Emerald City is assumed rigid, one can instead consider only a few clusters of thrusters, where each cluster is represented as if it is a single thrust force obtained as the resultant force of hundreds of thrusters each producing identical thrust-time histories in a given cluster location. Thus, each cluster location acts as one input in the control study. After finding the total thrust-time history at a particular location through the control study, one can then apportion equally the required thrust to individual thrusters. Thus, for given thrust capabilities of thrusters, the total number of thrusters required in a particular cluster can be found subsequently. For example, in Figure 6.5, only three cluster locations are shown as if there are only three control thrust inputs, F_1, F_2, and F_3, and the attitude control requirements are computed for that particular input configuration. How many individual thrusters may exist at each cluster becomes immaterial at this point and the control study can proceed.

4.3 ATTITUDE CONTROL DESIGN METHODOLOGY

The methodology developed in this chapter can be used to study the attitude control of the Emerald City for a wide range of ship configurations, thruster configurations, and initial disturbances. Two particular scenarios were examined and the control system

requirements were determined using the methods provided above. The results for particular mission studies can be found in Chapter 6. The outline of the methodology as described above is reviewed here in step-by-step form.

1. Choose a cluster configuration, characterized by Equation 4.9, to be studied and determine the slenderness ratio, r, from the ship configuration. Select an initial mean disturbance to investigate.
2. Use MATLAB (or a similar linear quadratic control regulator program) to determine the feedback gain, G, and the state response, x, to Equation 4.1 that minimizes the control design performance measure of Equation 4.3.
3. Use MATLAB to find the control torque, T, and the resultant cluster inputs, F (via Equation 4.14), versus time profiles, the non-dimensional power parameter, S^*, and observe the non-dimensional control time, τ_c, from the x response plots.
4. The resultant cluster thrusts required, F_i, for the control system can be found from F and dimensionalizing according to Equation 4.12. The root-mean square power and propellant required can be found from Equations 4.15 and 4.16, respectively.

One of the more important results obtained from the attitude control study was that the root-mean-square power required to damp out a disturbance is proportional to the square of the spin rate,

as shown in Equation 4.15. However, the torus diameter, D_t, can be written in terms of the spin rate and the g-level to be provided (see Chapter 5) as

$$D_t = \frac{n_g\, g}{2 n_s^2}\left(\frac{60}{2\pi}\right)^2 \tag{4.17}$$

where the spin rate n_s is now in revolutions per minute (rpm's). Inserting this equation into Equation 4.7 and converting the spin rate to rpm's yields

$$P_{rms} = \left(\frac{2\pi}{60}\right)^4 \frac{v_{ex}\, I_z\, n_s^4}{n_g\, g} \sqrt{\frac{S^*}{\tau_c}}\,. \tag{4.18}$$

Equation 4.18 clearly shows the benefit of keeping the spin rate below 1 rpm (since for $n < 1$, $n^4 < n$) in order to decrease the power requirements for attitude control. This result was taken into account in human factors studies when the torus was designed (see Chapter 5).

4.4 CONCLUSION

This year, the work on the dynamics and control aspect of Project WISH focused on determining the attitude control system requirements. A methodology was established to perform attitude control studies for a wide variety of missions. This methodology, which is summarized above, was used to design the attitude control system for the two missions of the Emerald City. The results of these mission designs can be found in Chapter 6.

CHAPTER 5

HUMAN FACTORS

5.0 INTRODUCTION

Maintaining a habitable environment in space presents problems with varying degrees of difficulty depending upon its size and how long that environment is to be maintained. Something such as food supply, which is a relatively minor concern on a shuttle mission, becomes a problem that could dominate the design of a spacecraft with a crew of between 500 and 1000 and an operational lifetime of fifty years. Because of the magnitude of the Emerald City, human factors will play an important role in its design. This chapter covers three topics that relate directly to the design of the spacecraft: the life support system, the need for artificial gravity, and the problem of radiation.

5.1 LIFE SUPPORT SYSTEM

Last year, the different functions of life support were approached using mechanical systems[2]. Separate systems were used for air revitalization, temperature and humidity control, waste management, and so forth. However, there are two major faults with using mechanical systems. Because of the three year plus mission times expected for Emerald City, all of these systems would have to run (unrealistically) with one-hundred percent efficiency. This is necessary to prevent compounds such as air or water from being recombined into products that cannot be recycled. More importantly

though, is that since these systems are mechanical, they are subject to breakdown and will eventually need spare parts. Storing these parts or making provisions for their manufacture would waste valuable space and add extra mass to the ship. For these reasons, another concept was explored for the primary life support system: biospheres.

A biosphere is a totally enclosed ecological system, with energy as its only input. The collective biomass of the earth is referred to as Biosphere I. Since all air, water, and waste is recycled internally, life can be sustained indefinitely. Clair Folsome created the first human-made biosphere inside a lab flask[10]. The flask contains only simple organisms, but it has been alive since it was sealed in 1968. What is proposed for Project WISH is that a scaled and more efficient version of Biosphere I be used as the primary life support system. Research is already being done to create biospheres large enough for human occupation. The Biosphere II project[10] in Oracle, Arizona has constructed an environmental facility sealed off from the atmosphere that uses its land and water four and ten times as efficiently as Biosphere I[11]. Total recycling of all atmospheric carbon dioxide takes place in half a day instead of eight years. The 150,000 cubic meter facility is designed as an environment for eight people who were recently sealed inside for a two year study. This figures to roughly 19,000 cubic meters per person. Even though this number would probably shrink if research were devoted to reducing the volume needed per person, it was used for sizing the crew section

to allow extra space for manufacturing, food processing, or other needs that have not yet been considered. The equation for the volume required in the torus section was then given by

$$V_{torus} - 19000N \tag{5.1}$$

where N is the number of crew members on the Emerald City. Research is currently aimed at understanding more about Biosphere I and to possibly extend man made biospheres onto other planets. It was believed that the concept of biospheres could legitimately be considered for a spacecraft with the life expectancy and magnitude of Project WISH.

5.2 ARTIFICIAL GRAVITY AND SHIP DESIGN

Of the little information known about long term effects of zero-g on the body, none of it is very positive. Without gravity, bones decalcify and muscles weaken and atrophy. A condition that could be described as "space anemia" also occurs[12]. This is due to a drop in the production of red blood cells. This otherwise harmless condition could pose a serious health problem if an astronaut were injured or became ill. To counter these effects, the crew section of Emerald City was designed as a torus rotating about a central hub to provide some level of artificial gravity. Even though providing 1 g would eliminate these effects, considerable power and mass savings can be made if an acceptable lower g-limit can be found. The relationship between rotation rate, distance from the spinning axis, and g-level is shown in Figure 5.1. Since the volume requirement (from Section 5.1) and shape for

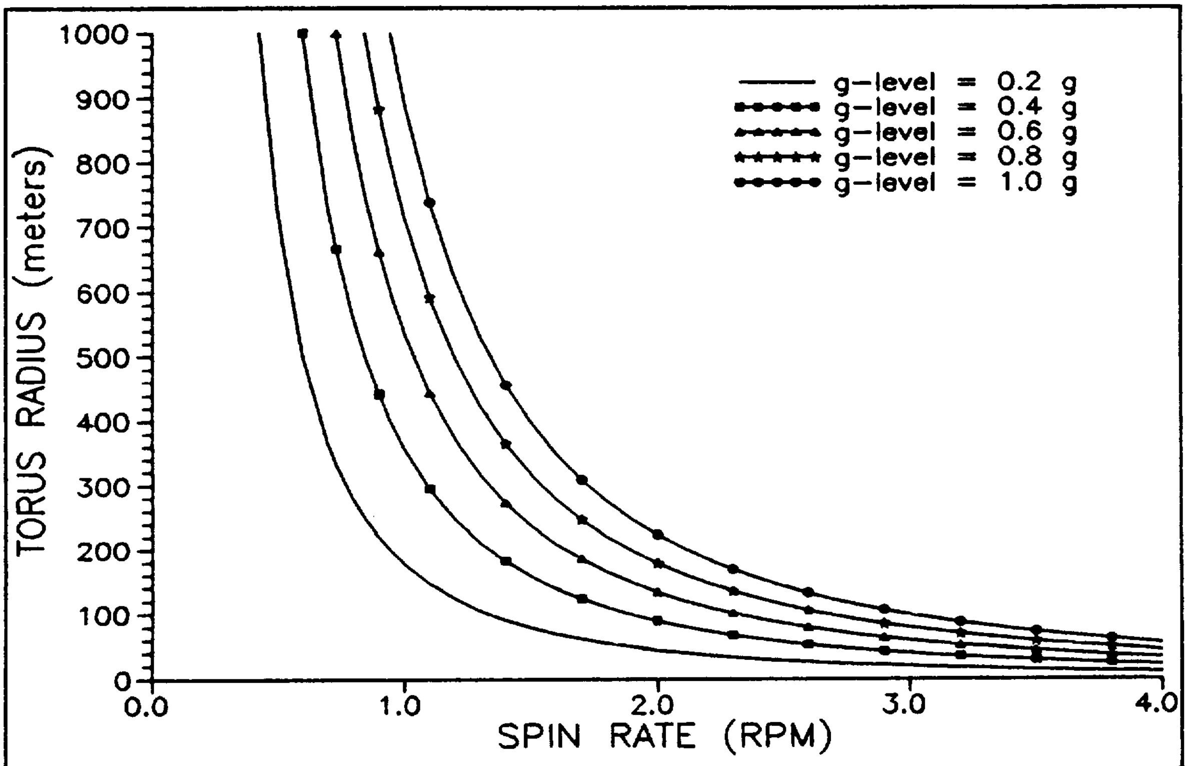

Figure 5.1 - Radius vs. Spin Rate for Different Gravity Levels

the crew section is known, the actual geometry is determined from the g-level and spin rate desired. Several possible geometries are listed in Table 5.1. Selection of the major radius of the torus, R (see Figure 6.1), is dependant on the constraints set for g-level and spin rate. One revolution per minute is used as an upper limit on spin rate because the power needed to maintain the attitude of the Emerald City is directly proportional to spin rate to the fourth power. This is discussed in greater detail in Chapter 4. Another reason behind this limit is human endurance. Most humans cannot endure extended periods of rotation at rates higher than 1 rpm[12]. Although there is no universal agreement on this limit, 1 rpm was chosen as it was the most conservative limit found. To

conserve mass, providing a g-level of only .8 g in the torus was considered. This lower gravity would hopefully have only minor effects on the crew. Even lower gravity levels could be used, which would reduce the mass further, but more information on the effects of long term exposure to lower gravity levels is needed before a lower g-limit can be set.

Table 5.1 - Possible Torus Geometries

Crew of 1000						
major radius (m)	minor radius (m)	%change in G top-bot	outer radius (m)	n for .8g (rpm)	n for .7g (rpm)	n for .6g (rpm)
500	44.2	17.7	544	1.15	1.07	0.99
540	42.5	15.7	582	1.11	1.04	0.96
580	41.0	14.1	621	1.07	1.00	0.93
620	39.6	12.8	660	1.04	0.97	0.90
660	38.4	11.6	698	1.01	0.95	0.88
700	37.3	10.7	737	0.99	0.92	0.85
Crew of 500						
major radius (m)	minor radius (m)	%change in G top-bot	outer radius (m)	n for .8g (rpm)	n for .7g (rpm)	n for .6g (rpm)
500	31.2	12.5	531	1.16	1.09	1.01
540	30.0	11.1	570	1.12	1.05	0.97
580	29.0	10.0	609	1.08	1.01	0.94
620	28.0	9.0	648	1.05	0.98	0.91
660	27.2	8.2	687	1.02	0.95	0.88
700	26.4	7.5	726	0.99	0.93	0.86

Another factor that contributes to the mass of the torus is the fact that it must hold an atmosphere. If a stressed skin is used, the necessary skin thickness and structural mass (M_{ss}) of a torus, with major radius R and minor radius r, is found using the formulas shown on the top of the next page[13]. The first two formulas find the skin thicknesses needed to withstand the principal stresses on the surface of the torus. Generally, t_{hoop} is greater than $t_{meridional}$, so t_{hoop} is used to find the structural mass. Structural mass is calculated for several configurations later in this chapter using the working stress (σ_w) and density (ρ) of

$$t_{meridional} = \frac{P_{atm} r}{\sigma_w}$$

$$t_{hoop} = \frac{\frac{P_{atm}}{2}\frac{r}{R} + \frac{P_g}{\pi}}{\sigma_w - \rho R} R \tag{5.2}$$

$$M_{ss} = 4\pi^2 r R t_{hoop} \rho$$

aluminum. P_g is the pressure due to the load distribution inside the torus. One way to reduce the structural mass is to lower the atmospheric pressure (P_{atm}) inside. Doing this would also result in effects similar to living at higher altitudes on Earth: the coughing mechanism would be less effective, a normal speaking voice would not carry as far, boiling temperature would be lowered, etc. Since structural mass is relatively minor compared to the whole ship, a pressure of one atmosphere is used in the crew section for all calculations.

5.3 RADIATION AND SHIELDING

The principle risk of living in space is from radiation induced cancer. The National Council on Radiation Protection and Measurements[14] has recommended a career limit for whole body exposure with a lifetime three percent excess risk of fatal cancer. Figure 5.2 is based on age at the start of exposure with a ten year career assumed. This recommendation, however, is made not for exploratory missions, such as a Mars mission or those that would be undertaken by the Emerald City, but for multiple missions of approximately 90 days over a 10 year period. What is needed for

Project WISH is a limit for constant exposure. Table 5.2 shows the limits recommended for constant public and occupational exposure in the United States. The occupational annual exposure limit of 5 rem per year would be the limit used on the Emerald City.

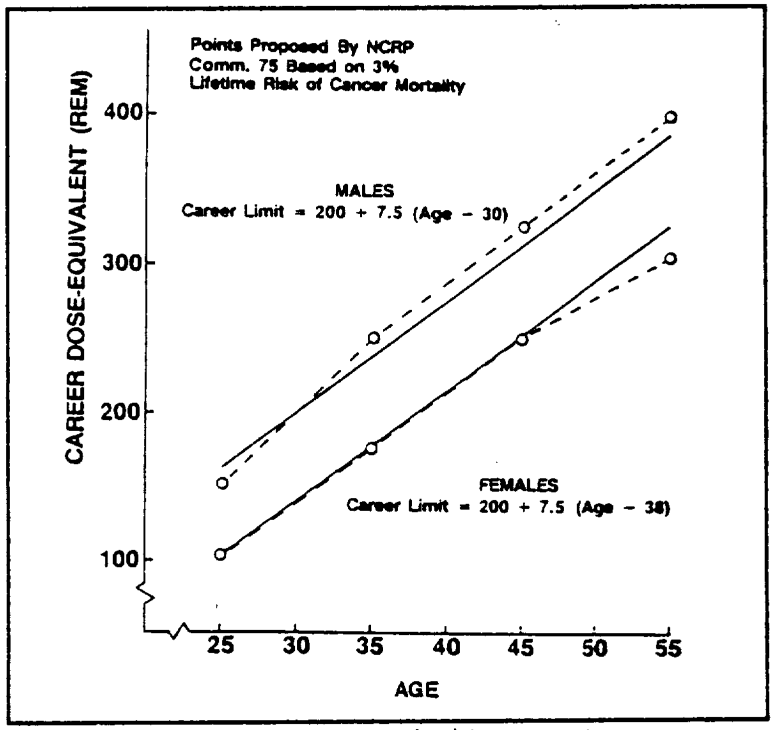

Figure 5.2 - Career Limit vs. Age

There are four major sources of radiation for Project WISH: galactic cosmic radiation, radiation from solar flare events, the propulsion system, and the power system. Of these four, only the last two can be reduced by moving the source away from the crew or by covering the source with a shield. Cosmic rays are present throughout the solar system at relatively constant intensities, decreasing only during solar flare activity. Unshielded dose rates from cosmic rays are estimated to be from 30 to 50 rem per

Table 5.2 - Continuous Dose Limits

Occupational	
Annual	5.0 rem
Lifetime (guideline)	Age*1.0 rem
Planned or special emergency	10.0 rem
Public	
Annual, continuous	0.1 rem
Annual, occasional	0.5 rem

year[15], while large solar flares are capable of delivering doses in excess of 1000 rem in only a week[16]. Since shielding these sources is impossible, the only way to protect from them is to shield the crew section. Radiation from the power system and the gaseous uranium core of the propulsion system can be attenuated by separating them from the crew and by partial shielding (see Figure 5.3). Unfortunately, the expanding plume of exhaust from the gas-

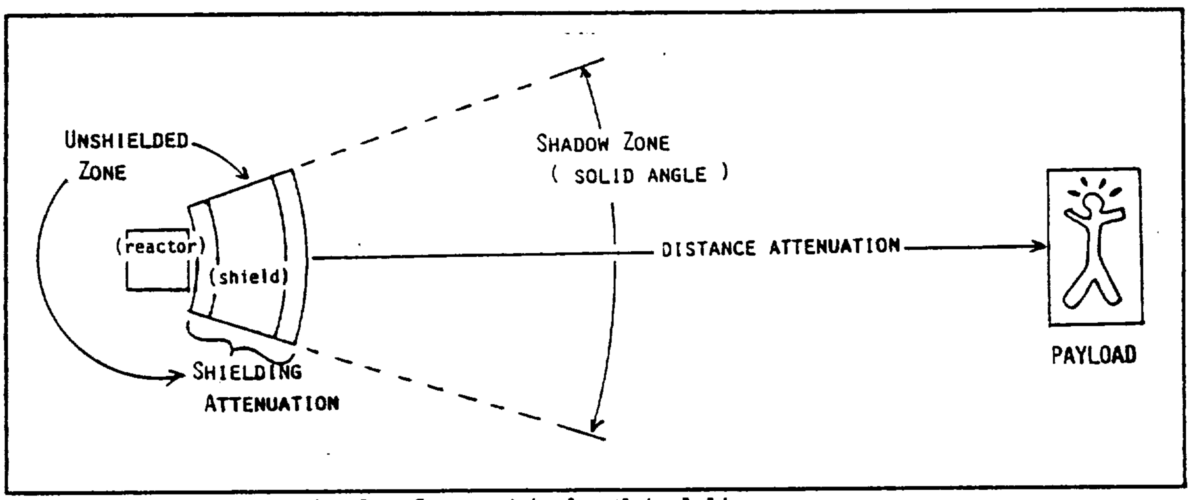

Figure 5.3 - Method of Partial Shielding

core rocket contains fission fragments that emit gamma radiation back towards the crew section. Increasing the distance between the crew and the nozzle exit will reduce the dose rate, but structural limits will only allow so much separation. The only other means of attenuation is to shield the crew section. The actual dose rate was calculated with the parameters of the gas-core engine from Chapter 3 using the formulas given by Charles Masser[17]. It is primarily dependant on the amount of separation between the crew and the engine, and the amount of time that fission fragments

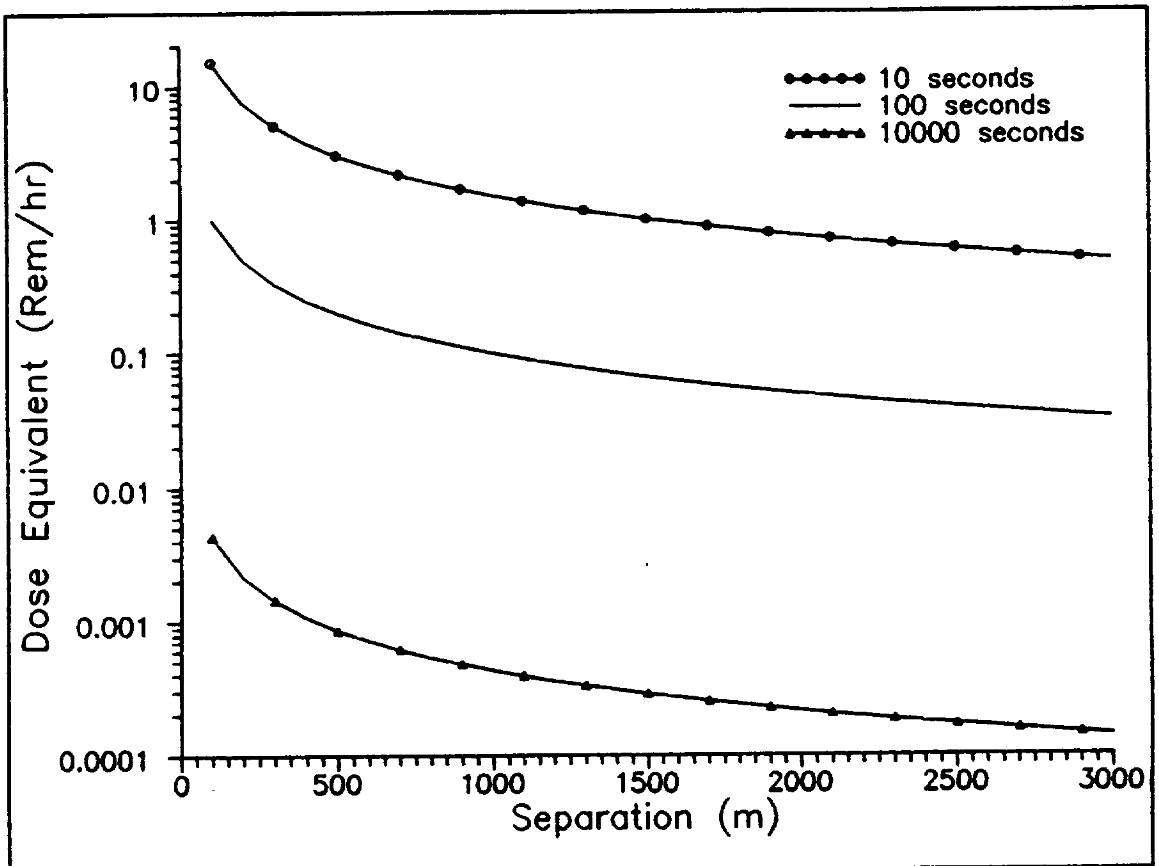

Figure 5.4 - Dose Rate vs. Separation for Varying Fission Fragment Retention Times

remain in the core before exiting the nozzle. Figure 5.4 shows this relationship for three different fission fragment retention times. The only estimate for the fission fragment retention time of the gas-core engine comes from Masser, who estimates it to be about 100 seconds. Note that the numbers in Figure 5.4 are only for one engine. It has been assumed that for the case of multiple engines, the dose rate will be additive. Although this assumption is probably not accurate, it is one that had to be made because of the limited knowledge of radiation by the members of this project.

Galactic cosmic radiation is considered to be the primary source of radiation because it is always present and cannot be

shielded at the source. No matter what their intensity, solar flares and engine exhaust will only be present for periods of about a week or two at a time. Because of this, the selection of a primary form of shielding was based on protecting against galactic cosmic radiation. The most common method of shielding is passive shielding. Passive shielding, known to be effective, simply puts a physical barrier between the source and the target. A more theoretical type of shielding is active shielding. It uses either an electromagnetic or electrostatic field to repel incoming radiation. A drawback common to both field types is that, depending on how much power they receive, they only shield radiations up to a certain energy level. Electrostatic fields were eliminated because a potential on the order of 3 GV is needed on the surface of the torus[18]. This potential would limit extra-vehicular activity and also generate lethal bremsstrahlung radiation fields inside the torus. An electromagnetic shield was rejected because higher energy radiation could become trapped inside the field. This would create something similar to the Van Allen belts around the Earth only much more dangerous considering the smaller distances involved. Another common failing of the shields is that they only stop charged particles. Gamma rays and neutrons, the more destructive types of radiation, are not affected.

With only passive shielding remaining, materials that could best attenuate radiation and still save mass were explored. Townshend compares different materials and finds liquid hydrogen to be

very effective[19]. Liquid hydrogen is more effective than other materials because it contains no neutrons for the incident radiation to scatter upon collision. Table 5.3 shows that 100 grams per square centimeter will reduce the dose to below 5 rem per year. One hundred grams per square centimeter of liquid hydrogen corresponds to a shield thickness of 14 meters. The mass of this type of shield, as well as the structural mass, is found for four different torus configurations in Table 5.4. Notice that even if the crew size is cut in half, the masses for the torus do not significantly drop. This is because of the geometrical nature of a torus, a large volume change results in a relatively minor change in surface area.

5.4 CONCLUSION

Even though all the considerations in this chapter have not been solved, enough has been determined to create a pattern that can be used to make an initial design for the Emerald City. The results of research in the areas of biospherics, microgravity, radiation, and other fields not yet considered will set the limits that help finalize any design. The information presented in this chapter will be used for a preliminary design of the Emerald City for two particular missions, which may be found in Chapter 6.

Table 5.3 - Solar Minimum Galactic Cosmic Ray Depth Dose Equivalent in Tissue as a function of Particle Type and LH_2 Shield Thickness

thickness (gm/cm^2)	dose equivalent, rem/yr, from				
	neutrons	protons	alpha's	HZE	total dose
	skin dose equivalent (0 cm depth)				
0.0	0.0	9.4	6.7	101.6	117.7
3.0	0.2	6.6	2.7	31.8	41.3
10.0	0.6	7.8	1.5	6.3	16.2
25.0	0.8	8.1	0.4	0.4	9.7
50.0	0.7	6.6	0.1	<.1	7.4
75.0	0.6	4.8	<.1	<.1	5.4
100.0	0.4	3.3	<.1	<.1	3.8
	BFO dose equivalent (5 cm depth)				
0.0	1.4	8.0	2.9	43.0	61.1
3.0	1.8	8.8	2.2	21.2	34.1
10.0	1.9	9.6	1.2	4.4	17.2
25.0	1.7	9.4	0.1	0.3	11.7
50.0	1.3	7.4	<.1	<.1	8.7
75.0	0.9	5.3	<.1	<.1	6.2
100.0	0.7	3.6	<.1	<.1	4.3

Table 5.4 - Structural and Shield Mass Estimates

crew size	major radius	minor radius	surface area	shield mass	structural mass	skin thickness
	(m)	(m)	(m^2)	(kg)	(kg)	(cm)
1000	500	44.2	8.725e+05	8.725e+08	4.101e+07	1.741
1000	700	37.3	1.031e+06	1.031e+09	5.052e+07	1.815
500	520	30.6	6.282e+05	6.282e+08	2.407e+07	1.419
500	700	26.4	7.296e+05	7.296e+08	3.027e+07	1.537

CHAPTER 6

REPRESENTATIVE MISSION DESIGN

6.0 INTRODUCTION

Once the methodologies of the previous chapters have been formulated, two sample missions were examined in order to obtain some preliminary sizing and mass estimates for the Emerald City. A Saturn Envelope mission and the currently popular Earth-to-Mars mission were chosen as representative scenarios for this preliminary design. The design for these missions were determined by incorporating the orbital mechanics, propulsion, attitude control and human factors subsystems. Orbital mechanics (Chapter 2) was used to determine the ΔV's required for the selected missions. The propulsion system study (Chapter 3) was used to determine the number of engines and the total engine and propellant masses. These were then used to size the Emerald City in terms of volume of propellant, engine spacing and location, length of central pole, etc. Human factors (Chapter 5) was used to determine the sizing of the torus, and the attitude control study (Chapter 4) was used to calculate the control thruster requirements. With all of the work laid down in the previous chapters in mind, it is now possible to proceed with the design of both missions.

6.1 PROCEDURE

The procedure that was followed in the design of these missions is listed sequentially as follows.

1. Obtain the ΔV required for each mission from orbital mechanics.
2. Set values for the specific impulse, I_{sp}, and the thrust per engine, F_i, for the propulsion system. The mass of each engine, m_{wi}, is then found from the computer program of Appendix B.
3. Determine the torus mass, m_{torus}, from the masses of the biosphere, the shield, and the torus structure. These masses are dependant upon the number of crew members selected. The dry mass, m_{dry}, is then estimated by making allowances for the engine, cargo, and remaining subsystem masses.
4. Calculate the number of engines required and the total engine mass, m_w, for each mission.
5. Calculate the payload mass, m_l, for the missions. This mass is defined as $m_{dry} - m_{torus} - m_w$ and includes cargo as well as other systems that have yet to be sized (heat rejection, power, tankage, etc.).
6. Calculate the total initial mass, m_0, and the propellant mass, m_p. The propellant mass ratio, m_p / m_0, and the payload mass ratio, m_l / m_0, are then found.
7. Calculate the volume of propellant required for the mission, V_p. Then, by estimating a stem radius, r_p, the height of propellant, and hence the minimum stem length, h_p, can be found.

Neptune and Pluto cannot be reached at all in flight times of up to five years with this ΔV ceiling. It should be noted that many of the transfers to Saturn can be made with ΔV's considerably less than 50 km/s round trip, but for this mission, it was assumed that the Emerald City was in a poor orientation (ΔV-wise) with Saturn and that a three year flight time was essential. A possible motivation for such a mission might be that the Emerald City is required to support the first manned exploration of Saturn's moon Titan, and that the planetary alignments and limitations on the interplanetary transfer ship require a particular launch date from the Martian colony and arrival time at Saturn.

For this mission, the individual ΔV's and the powered-flight times required for the four separate burns were not considered, but only the total ΔV and powered-flight time. The number of engines was found based on these total values and a given total powered-flight time of 20 days, but it should be kept in mind that the powered-flight times would not be the same for all four burns, even if the ΔV required would be the same. This is because the mass of the Emerald City is smaller for each of the burns, and hence the longest powered-flight time would be the first one.

The results for the Saturn mission can be found in Tables 6.1. Some important values to note are the total initial mass, 4.16×10^9 kilograms, the number of engines, 172, the torus radius, 700 meters, and the height of the pole section, 1270 meters. The payload mass ratio for this mission was found to be 0.083, or 8.3% of the total initial mass is payload. It must be remembered,

however, that the payload mass as defined here is the mass of the cargo plus the mass for any other systems that were not considered during Phase II (power systems, thermal rejection systems, tankage mass, etc.).

6.2.2 Earth-to-Mars Mission

The second mission that was examined was a transfer from 1 AU to Mars. This mission represents a more common mission that the Emerald City might be required to undertake, perhaps carrying new colonists and supplies that had been shuttled to the Emerald City, just outside the Earth's sphere of influence, to the ever expanding Martian colonies. For this mission, the ΔV required was broken down into the departure and arrival ΔV's. The mission was scheduled to leave Earth on November 7, 2050 and the flight time was determined by minimizing the ΔV for this launch date using the MULIMP program. The ΔV's were found to be 5.1 km/s and 7.5 km/s for a total ΔV of 12.6 km/s, and the flight time was found to be 1.52 years. The first powered-flight time was set at 5 days and the number of engines required was determined. Then, keeping this number of engines constant, the second powered-flight time was found. From Table 6.1, it can be seen that the results of this were found to be 33 engines, a 5 day initial powered-flight time, and a 6.53 day terminal powered-flight time. Some of the other important design values for this mission are the initial mass, 1.295×10^9 kg, the torus radius, 700 meters, and the pole height, 569 meters.

Table 6.1 - Summary of Design Variables for Sample Missions

	Saturn Mission	Earth to Mars
People	1000	500
ΔV_{total} (km/s)	50	12.6
ΔV_1 (km/s)	-	5.1
ΔV_2 (km/s)	-	7.5
F_i (Newtons)	4.4×10^5	4.4×10^5
I_{sp} (seconds)	5000	5000
$t_{pr,total}$ (days)	20	11.53
$t_{pr,1}$ (days)	-	5
$t_{pr,2}$ (days)	-	6.53
number of engines	172	33
m_{dry} (kg)	1.5×10^9	1×10^9
m_{torus} (kg)	1.111×10^9	7.746×10^8
m_w (kg)	4.4376×10^7	8.514×10^6
m_1 (kg)	3.457×10^8	2.165×10^8
m_p (kg)	2.658×10^9	2.947×10^8
m_0 (kg)	4.16×10^9	1.295×10^9
m_1 / m_0	0.083	0.167
m_p / m_0	0.639	0.228
V_{LH2} (m^3)	3.742×10^7	4.151×10^6
r_{pole} (m)	100	50
h_p (m)	1200	528
h (m)	1270	569
R (m)	700	700
r_{min} (m)	37	26
I_z (kg m^2)	5.6144×10^{14}	3.8061×10^{14}
I_x, I_y (kg m^2)	9.9568×10^{14}	2.1731×10^{14}
r (I_z / I_x)	0.563876	1.75154
max/min g-levels	0.8/0.72	0.8/0.74
n (spin rate, rpm)	0.99	0.99

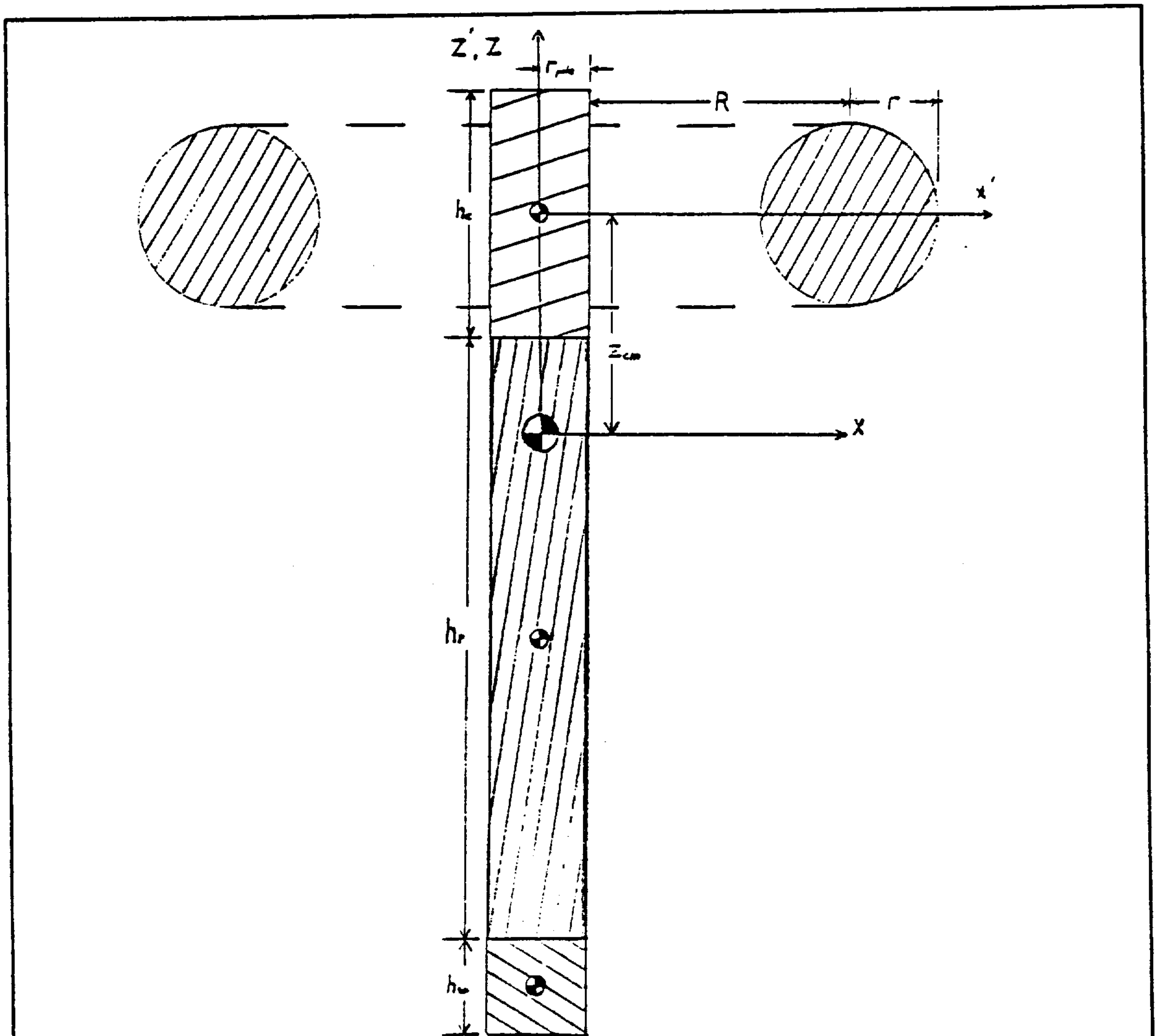

Figure 6.1 - Schematic of the Emerald City

6.2.3 Attitude Control System Requirements

An attitude control system was designed for the two particular missions examined for a sample initial disturbance and a sample thruster configuration. Using the methodology developed in Chapter 4, the root-mean-square power, propellant mass, maximum thrust, and control time was determined, as may be seen in Table 6.2. Also, the state response, control torque profile, and thrust profile for

the Saturn Envelope mission configuration are shown in Figures 6.2-6.4 for the sample thruster configuration shown in Figure 6.5.

Table 6.2 - Attitude Control System Requirements

I_z / I_x	0.56	1.75
Initial Disturbance	$\left[\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2\right] =$	$\left[.1 \quad .1 \quad .1n \quad .1n\right]$
P_{rms} (watts)	6.176×10^{12}	7.86×10^{11}
$m_{p,control}$ (kg)	2.105×10^{7}	3.458×10^{6}
$F_{control,max}$ (N)	2.988×10^{9}	1.911×10^{9}
$t_{control}$ (s)	~ 100	~ 100

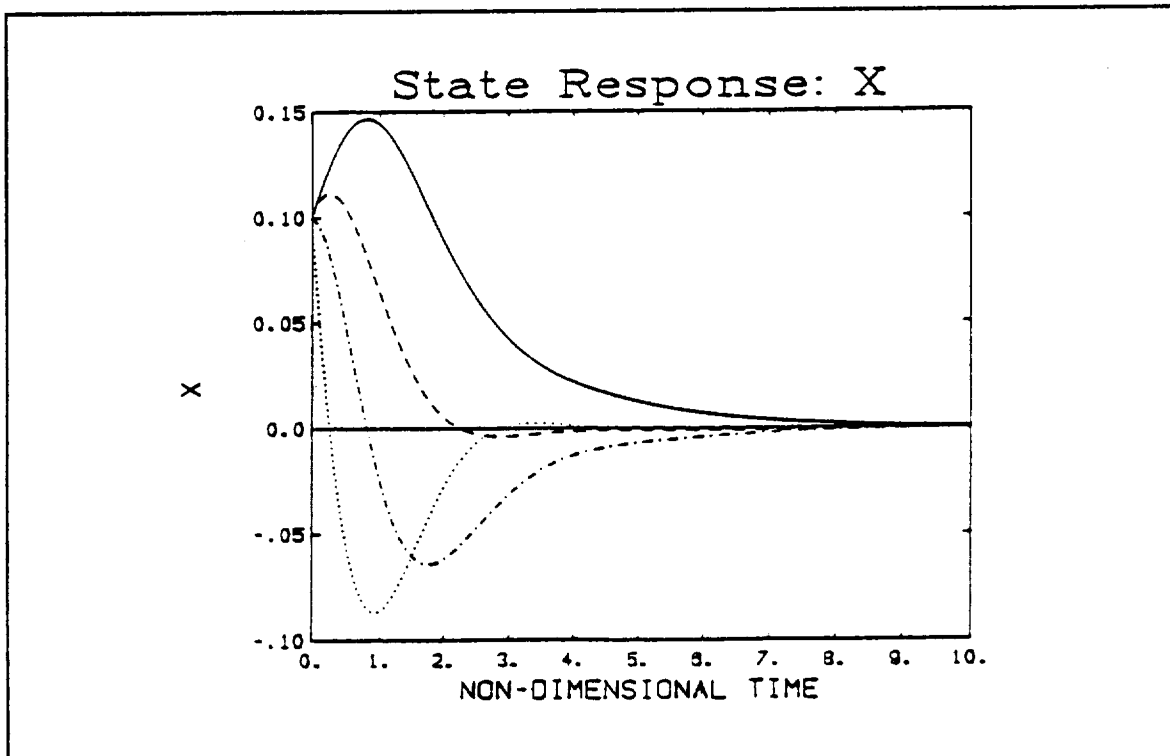

Figure 6.2 - Non-Dimensional State Response (r = 0.5)

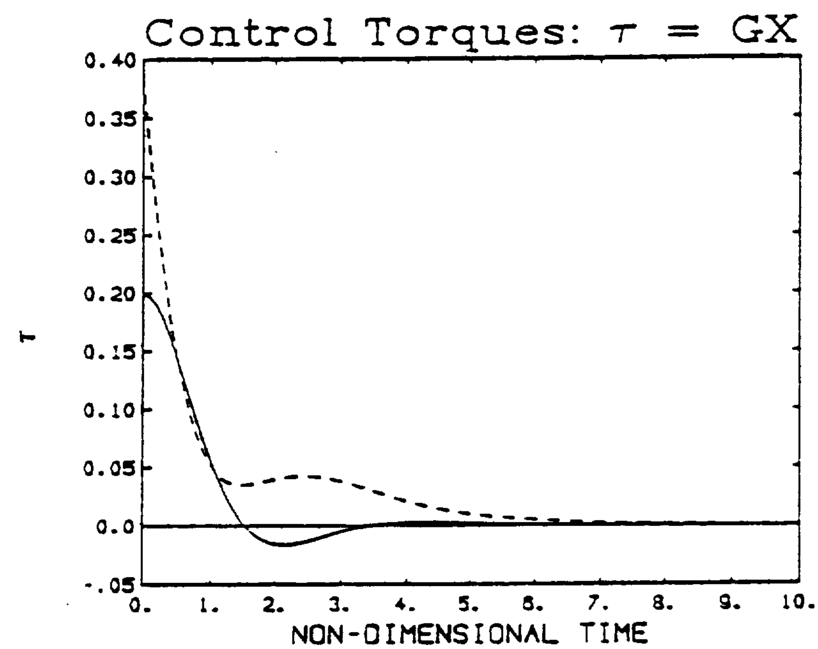

Figure 6.3 - Control Torque Profile (r = 0.5)

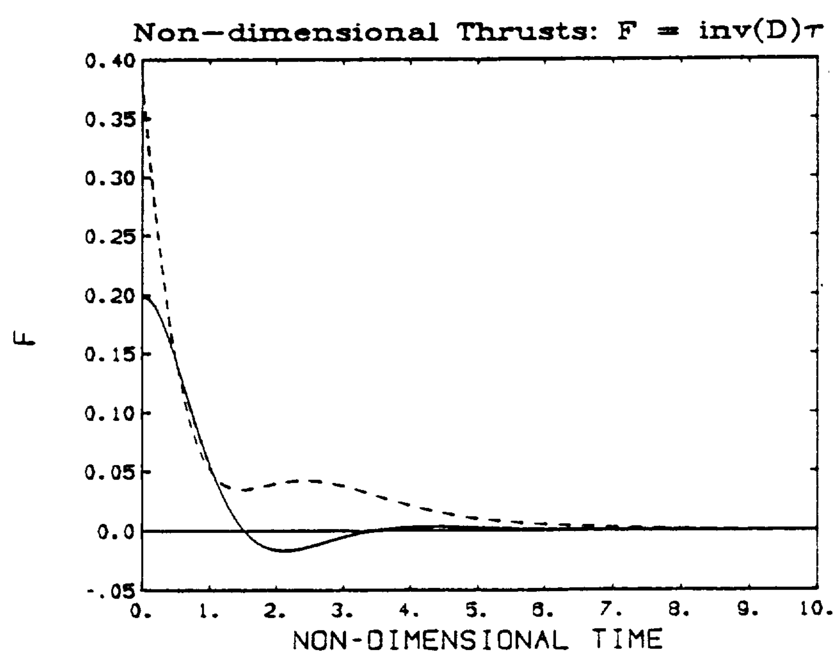

Figure 6.4 - Non-dimensional Thrust Profile (r = 0.5)

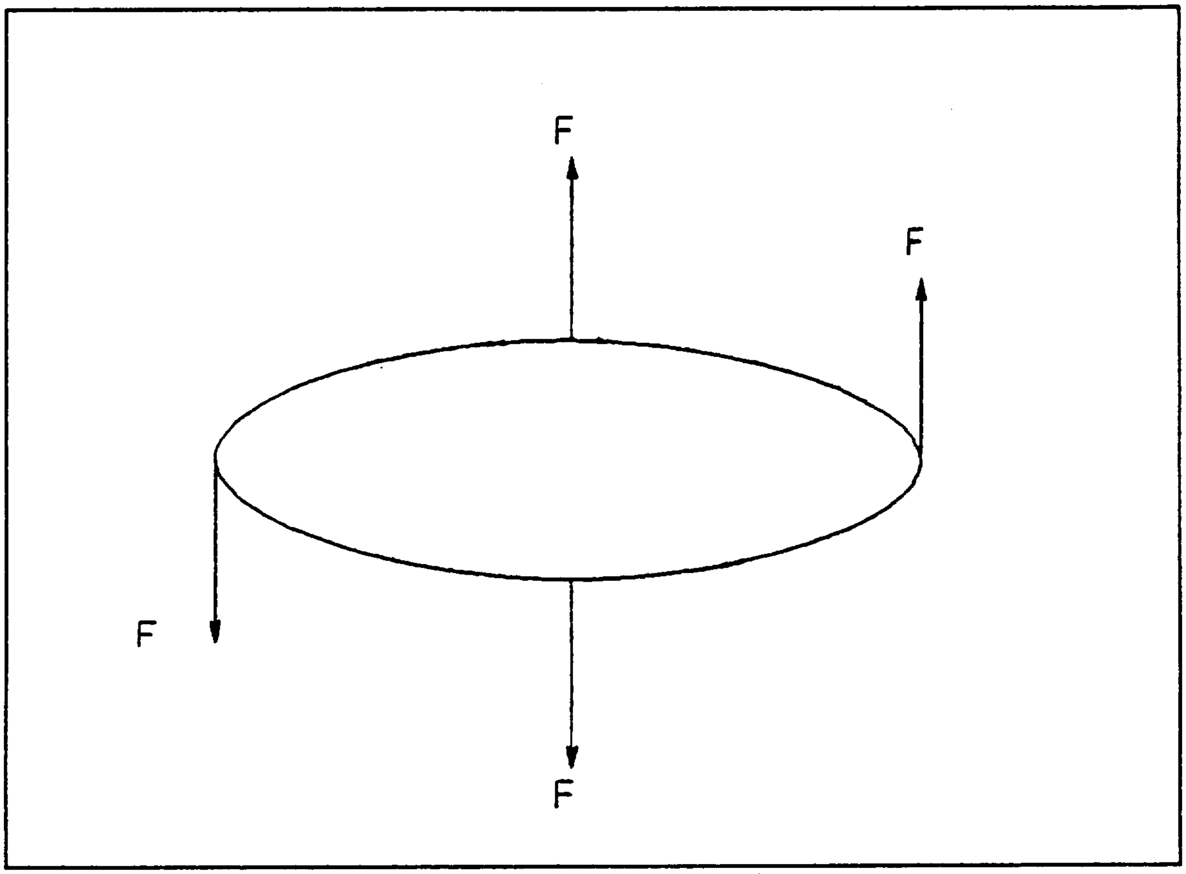

Figure 6.5 - Sample Thruster Configuration

6.3 CONCLUSION

In this chapter, two particular missions were examined for the Emerald City. However, using the methodology established here, a wide variety of missions may be looked at and the configuration for the Emerald City may be determined. In the future, different mission parameters, such as crew size, ΔV requirements, sample initial disturbances, attitude thruster configurations, etc..., may be examined. It is felt that this methodology is the main accomplishment of the Phase II design of Project WISH.

CHAPTER 7

CONCLUSION

7.0 PROJECT SUMMARY

Phase II of Project WISH was able to accomplish a great deal towards the design of the Emerald City. In orbital mechanics, a nominal orbit of 4 AU's was selected, and the operational envelope for the Emerald City was determined. An open-cycle gas-core nuclear thermal propulsion system was chosen as the main propulsion unit, and a detailed study was conducted on this type of engine. For attitude control, a methodology was established to determine the thruster requirements needed to damp out a disturbance experienced by the Emerald City. Human factors looked at the problems of life support, artificial gravity, and radiation shielding. Biospheres were examined as the primary method to sustain life, while the torus size requirements were determined for the different levels of gravity that the crew would need. Radiation shielding proved to be one of the major design problems encountered, and mass estimates were obtained for shielding from cosmic radiation. The ultimate result of the Phase II study was the generation of Tables 6.1 and 6.2. These tables incorporated information from all of the subsystems studied, and provide data on the estimated size and mass requirements of the Emerald City. By examining these figures, one can realize the magnitude of a ship such as the Emerald City.

concessions to the unknown, we had best frankly recognize that irrational faith in the value of a new capability often plays an essential part in the early evolution of a new concept. Certainly, at that point, the lack of a recognized application is a demonstrably unsound basis for the concept's rejection."

Table 7.1 - Envisioned Time Line for Project WISH

1989	President Bush announces Mars Initiative to reach the Red Planet in 30 years.
1991	NASA presents a program to send sensing probes throughout the solar system. Projects include the return to the inner solar system, exploration of the asteroid belt, and further missions to the outer planets.
1998	Space Station Freedom becomes operational.
2001	Heavy Lift Launch Vehicle makes its maiden flight.
2004	Construction begins on a near-geosynchronous Earth orbiting space station.
2009	U.S. returns to the moon.
2011	- Construction begins on the Moon Base. - Maiden flight of National Aerospace Plane.
2015	Moon Base becomes fully operational.
2019	First manned mission to Mars.
2023	- First living modules constructed on Mars. - Construction begins on Reusable Interplanetary Ships (R.I.S.'s) for carrying personnel and cargo.
2028	Mars Base becomes fully operational.
2040	Implementation of Project WISH.
2050	The Emerald City becomes operational.

APPENDIX A

EQUATIONS FOR ΔV MINIMIZATION PROBLEM

The objective function given for the ΔV minimization problem set up in Chapter 2 was given by Equation 2.4

$$\Delta V_T = \Delta V_1 + \Delta V_2 = \vec{V}_1 - \vec{V}_o + \vec{V}_f - \vec{V}_2 \tag{2.4}$$

where ΔV_1 and ΔV_2 are the velocity changes along the thrust vector direction, which will be kept constant during each powered-flight phase. The constraints were given in Chapter 2 as

1. $t_{ff} + t_{pr1} + t_{pr2} < TOF_{max}$
2. $a_0 < a_{0,max}$
3. $t_{pr1} + t_{pr2} < t_{pr,max}$
4. $x_{f,EC} = x_{f,TO}$.
5. $a_0,\ t_{pr1},\ t_{pr2},\ t_{ff} > 0$.

Thus, the problem is to determine the relationships between these values and the design variables of the problem.

Since a circular nominal orbit and target planet orbit has been assumed, v_0 and v_f are given by the formulas for circular orbital velocity,

$$v_0 = \sqrt{\frac{\mu}{r_0}} \tag{A.1}$$

$$v_f = \sqrt{\frac{\mu}{r_f}} \tag{A.2}$$

which are known for a given nominal orbit and target planet. However, finding the velocities v_1, the velocity at the end of the first powered-flight phase, and v_2, the velocity at the beginning of the second powered-flight phase, is more involved. To find the equations for these velocities, the kinematic relationship

$$dv = a \cdot dt \tag{A.3}$$

was integrated along the direction of thrust, with an acceleration term in this direction given by

$$a(t) = \frac{a_o}{1 - \frac{a_o t}{I_{sp} g}}. \tag{A.4}$$

This term arises from the fact that as the Emerald City is burning with a constant thrust, it is expending mass, and thus the acceleration will increase with time according to Equation A.4. Inserting Equation A.4 into Equation A.3 and performing the integration yields

$$v = v_i - I_{sp} g \ln(1 - \frac{a_o t_{pr}}{I_{sp} g}), \tag{A.5}$$

which becomes

$$\Delta V_1 = v_{1F} - v_{oF} = - I_{sp} g \ln(1 - \frac{a_o t_{pr1}}{I_{sp} g}) \tag{A.6}$$

for the first powered-flight portion of the transfer. Similarly, for the second powered-flight portion of the transfer, the velocity change is given by

$$\Delta V_2 = v_{fF} - v_{2F} = -I_{sp}g \ln(1 - \frac{a_{o2}\ t_{pr2}}{I_{sp}g})$$

where (A.7)

$$a_{o2} = \frac{a_o}{1 - \frac{a_o\ t_{pr1}}{I_{sp}g}}.$$

The subscript F in Equations A.6 and A.7 denotes the components of the respective velocity vectors projected along the thrust direction (it is assumed that the thrust angles, $\beta_{1,2}$, will not be changed during a powered-flight phase). It can easily be seen that by using Equations A.6 and A.7, the objective function of Equation 2.4 can be written as

$$\Delta V_T = -I_{sp}g[\ln(1 - \frac{a_o t_{pr1}}{I_{sp}g}) + \ln(1 - \frac{a_{o2} t_{pr2}}{I_{sp}g})]. \qquad (A.8)$$

where a_{02} is not an independent variable, but given in Equation A.7.

The application of the constraint functions needed many more equations and relationships. First, continuing in the same manner as was used to find Equation A.5, the relationship

$$dx = vdt \qquad (A.9)$$

was integrated with the results from Equation A.5 inserted for v, and the result was found to be

$$x_{1,f} = x_{o,2} + v_{o,2}t_{pr1,2} + \frac{(I_{sp}g)^2}{a_{o,o2}}\Phi_{1,2}$$

where (A.10)

$$\Phi_{1,2} = (1 - \frac{a_{o,o2}\ t_{pr1,2}}{I_{sp}\ g})[\ln(1 - (\frac{a_{o,o2}t_{pr1,2}}{I_{sp}\ g}) - 1] + 1.$$

Equation A.10 yields the position along the direction of the thrust vector and the subscript 1,f means at position 1 or f, etc. Thus, armed with Equations A.5, A.10, and some fundamental relationships from astrodynamics, the constraints can be related to the design variables of the problem in the following manner. First, the inertial X- and Y-components of the velocity and radius vectors at the end of the first powered-flight phase can be found by applying coordinate rotation matrices to Equations A.5 and A.10. These equations are given by

$$v_{1x} = -\sqrt{\frac{\mu}{r_o}}\ \sin\theta_o - I_{sp}g\sin(\beta_1 - \theta_o)\ln(1 - \frac{a_o t_{pr1}}{I_{sp}g})$$

$$v_{1y} = \sqrt{\frac{\mu}{r_o}}\ \cos\theta_o - I_{sp}g\cos(\beta_1 - \theta_o)\ln(1 - \frac{a_o t_{pr1}}{I_{sp}g}) \qquad \text{(A.11)}$$

$$v_1 = \sqrt{v_{1x}^2 + v_{1y}^2}$$

and

$$r_{1x} = r_o\cos\theta_o - \sqrt{\frac{\mu}{r_o}}\,t_{pr1}\sin\theta_o + \frac{(I_{sp}g)^2}{a_o}\sin(\beta_1 - \theta_o)\Phi$$

$$r_{1y} = r_o\sin\theta_o + \sqrt{\frac{\mu}{r_o}}\,t_{pr1}\cos\theta_o + \frac{(I_{sp}g)^2}{a_o}\cos(\beta_1 - \theta_o)\Phi \qquad \text{(A.12)}$$

$$\Phi = (1 - \frac{a_o t_{pr1}}{I_{sp}g})[\ln(1 - \frac{a_o t_{pr1}}{I_{sp}g}) - 1] + 1$$

$$r_1 = \sqrt{r_{1x}^2 + r_{1y}^2}$$

where the known variables are r_0 and I_{sp} and the design variables are θ_0, t_{pr1}, a_0, and β_1. Once the velocity and position vectors at the initial point of the free-flight trajectory (point 1) are known, the transfer orbit is defined by the following equations:

$$e_x = \frac{1}{\mu}[(v_1^2 - \frac{\mu}{r_1})r_{1x} - (r_{1x}\,v_{1x} + r_{1y}\,v_{1y})\,v_{1x}]$$

$$e_y = \frac{1}{\mu}[(v_1^2 - \frac{\mu}{r_1})r_{1y} - (r_{1x}\,v_{1x} + r_{1y}\,v_{1y})\,v_{1y}] \tag{A.13}$$

$$e = \sqrt{e_x^2 + e_y^2}$$

$$\gamma_o = \cos^{-1}(\frac{e_x}{e}) \quad (quadrant) \tag{A.14}$$

$$E = \frac{v_1^2}{2} - \frac{\mu}{r_1} \tag{A.15}$$

$$a = -\frac{\mu}{2E} \tag{A.16}$$

$$\gamma_1 = \cos^{-1}(\frac{a(1 - e^2) - r_1}{e\,r_1}) \quad (quadrant) \tag{A.17}$$

$$h = \sqrt{a(1 - e^2)\mu} \tag{A.18}$$

In the above equations, e is the eccentricity of the transfer orbit, τ_0 is the direction of the eccentricity vector, E is the specific energy of the transfer orbit, a is the semi-major axis of the transfer orbit, τ_1 is the true anomaly at the end of the first powered-flight time, and h is the angular momentum of the transfer orbit. Once the transfer orbit is defined through these equations, the variables at the terminal free-flight point (point 2) may be calculated. The design variable that was designated was γ_2, which is the true anomaly at point 2. This then defines the free-flight time of the transfer according to

$$t_{ff} = \sqrt{\frac{a^3}{\mu}}\ [2\pi k + (E_2 - e\ \sin E_2) - (E_1 - e\ \sin E_1)] \qquad \text{(A.19)}$$

where E is the eccentric anomaly, defined by

$$E_n = \cos^{-1}\left(\frac{e + \cos\gamma_n}{1 + e\cos\gamma_n}\right) \qquad \text{(A.20)}$$

and k is the number of times the periapsis is passed during the transfer. The rest of the values at point 2 are given by

$$r_2 = \frac{a(1 - e^2)}{1 + e\cos\gamma_2} \qquad \text{(A.21)}$$

$$v_2 = \sqrt{2\left(\mathrm{E} + \frac{\mu}{r_2}\right)} \qquad \text{(A.22)}$$

$$\theta_2 = \left[\frac{\gamma_o + \gamma_2}{360^\circ} - INTEGER\left(\frac{\gamma_o - \gamma_2}{360^\circ}\right)\right]\cdot 360^\circ \qquad \text{(A.23)}$$

$$\phi_2 = \cos^{-1}\left(\frac{h}{r_2 v_2}\right) \quad (quadrant) \qquad \text{(A.24)}$$

$$r_{2x} = r_2\cos\theta_2$$
$$r_{2y} = r_2\sin\theta_2 \qquad \text{(A.25)}$$

$$v_{2x} = -v_2\sin(\theta_2 - \phi_2)$$
$$v_{2y} = v_2\cos(\theta_2 - \phi_2)\ . \qquad \text{(A.26)}$$

The equations governing the final powered-flight phase of the trajectory are found by applying coordinate rotation matrices to Equations A.5 and A.10, similar to the way that Equations A.11 and A.12 were obtained. These equations were found to be

$$\begin{aligned} v_{fx} &= -v_2\sin(\theta_2 - \phi_2) - I_{sp}g\sin(\beta_2 - \theta_2)\ln(1 - \frac{a_{o2}t_{pr2}}{I_{sp}g}) \\ v_{fy} &= v_2\cos(\theta_2 - \phi_2) - I_{sp}g\cos(\beta_2 - \theta_2)\ln(1 - \frac{a_{o2}t_{pr2}}{I_{sp}g}) \end{aligned} \quad (A.27)$$

and

$$\begin{aligned} r_{fx} &= r_2\cos\theta_2 - v_2 t_{pr2}\sin(\theta_2 - \phi_2) + \frac{(I_{sp}g)^2}{a_{o2}}\sin(\beta_2 - \theta_2)\Phi_2 \\ r_{fy} &= r_2\sin\theta_2 + v_2 t_{pr2}\cos(\theta_2 - \phi_2) + \frac{(I_{sp}g)^2}{a_{o2}}\cos(\beta_2 - \theta_2)\Phi_2 \\ \Phi_2 &= (1 - \frac{a_{o2}t_{pr2}}{I_{sp}g})[\ln(1 - \frac{a_{o2}t_{pr2}}{I_{sp}g}) - 1] + 1. \end{aligned} \quad (A.28)$$

The above equations will play an important part in the determination of the constraints on the total time-of-flight and the rendezvous requirement. For the time-of-flight constraint, the constraint function is found by adding the two powered-flight times, which are design variables, to the free-flight time as found from Equation A.19. This value must then fall below the specified TOF_{max} (three years for most cases) for the constraint to be satisfied. For the rendezvous requirement, recall from Chapter 2 that the state vector at point i, x_i, was defined such that

$$\vec{x}_i = \{\vec{r}_i \quad \vec{v}_i\}. \quad (2.5)$$

The rendezvous constraint requires that $x_{f,EC} = x_{f,TO}$, which represents the Emerald City ending the transfer with the desired velocity and radius vectors at the target orbit. The state vector, $x_{f,EC}$, can be found from Equations A.27 and A.28, while the state vector, $x_{f,TO}$, can be determined from

$$\theta_{f,TO} = \theta_i + \sqrt{\frac{\mu}{r_p^3}}\ TOF \tag{A.29}$$

$$\begin{aligned} r_{x,TO} &= r_p \cos\theta_{f,TO} \\ r_{y,TO} &= r_p \sin\theta_{f,TO} \end{aligned} \tag{A.30}$$

$$\begin{aligned} v_{x,TO} &= -v_p \sin\theta_{f,TO} \\ v_{y,TO} &= v_p \cos\theta_{f,TO}. \end{aligned} \tag{A.33}$$

With the above formulation, care must be taken in the programming of the above equations to assure that the proper quadrant is used when inverse trigonometric functions are used.

APPENDIX B

ENGINE CHARACTERISTIC CALCULATIONS

A computer program was written to calculate the important engine parameters for an open-cycle gas-core nuclear engine. This program is listed below. The equations were either obtained from Ragsdale's paper or derived from basic considerations

```
*****************************************************************
      program gascore
*****************************************************************
* this program finds the parameters of a gas core rocket for a
* specific cavity diameter and Isp and prints the data to a file
* called engine.dat.               Enjoy.
*****************************************************************
      dimension f(5)
      open(unit=1,file="ENGINE.DAT")
      GRAV=9.80665
      PI=3.14159265
      ATM=1.01325E5
      UGC=8314
      THRUST=4.4E+5
      VUVC=.23
      write(6,10)
 10   format('1',5X,'ENTER SPECIFIC IMPULSE (Seconds)')
      read*,SPIMP
      write(6,40)
 40   format(5X,' ENTER REACTOR CAVITY DIAMETER (Meters)')
      read*,DIAM

**    Calculate critical mass of Uranium 233   **

      CD=3.3*DIAM
      CRITM=201.836-67.0167*CD+7.70629*CD**2-.158108*CD**3-
     &.0196678*CD**4+.00088141*CD**5

**    Calculate mass flow rate ratio        **

      DOTM=5.041871E-5*(VUVC**-10.799136)

**    Calculate hydrogen propellant temperature  **
      TH2=-294.461+4.36196*SPIMP+7.23731E-4*SPIMP**2-6.03953E-
     &7*SPIMP**3
      TH2=TH2+1.36261E-10*SPIMP**4-7.71588E-15*SPIMP**5

**    Calculate reactor cavity pressure        **
```

```
      PRES=(1066.57*CRITM*((THRUST*SPIMP)**.277)*(DOTM**.1012)/
     &(DIAM**3.277))**1.383

**    Calculate mass flow rate from nozzle     **

      PDOT=THRUST/(SPIMP*GRAV)

**    Calculate mass flow rate of hydrogen and uranium   **

      UDOT=PDOT/(DOTM+1)
      H2DOT=PDOT-UDOT

**    Calculate uranium temperature  **

      TU=140.06*((PRES*THRUST*SPIMP/DIAM)**.16)*(DOTM**.017)

**    Calculate pressure chamber volume and uranium volume   **

      VC=1.33333*PI*((DIAM/2)**3)
      VU=VUVC*VC

**    Calculate hydrogen ratio of specific heats in cavity   **

      GAMMAC=3.84502-.000336982*TH2+1.61689E-8*TH2**2-3.66763E-
     &13*TH2**3
      GAMMAC=GAMMAC+4.37181E-18*TH2**4-2.65532E-23*TH2**5+
     &6.49491E-29*TH2**6

**    Calculate hydrogen molecular mass in cavity   **

      H2MMC=2.32426-.000111528*TH2+2.53066E-9*TH2**2-2.49052E-
     &14*TH2**3
      H2MMC=H2MMC+8.94761E-20*TH2**4

**    Calculate throat conditions   **

      TTH=TH2/(1+((GAMMAC-1)/2))
      PTH=PRES/((1+(GAMMAC-1)/2)**(GAMMAC/(GAMMAC-1)))
      PTH=PTH/ATM
      GAMMAT=3.84502-.000336982*TTH+1.61689E-8*TTH**2-
     &3.66763E-13*TTH**3
      GAMMAT=GAMMAT+4.37181E-18*TTH**4-2.65532E-23*TTH**5+
     &6.49491E-29*TTH**6
      H2MMT=2.32426-.000111528*TTH+2.53066E-9*TTH**2-
     &2.49052E-14*TTH**3
      H2MMT=H2MMT+8.94761E-20*TTH**4
      ATH=PDOT*SQRT(TH2)/PRES
      ATH=ATH/SQRT(GAMMAT*H2MMT*(2/(GAMMAT+1))**
     &((GAMMAT+1)/(GAMMAT-1))/UGC)
      VTH=SQRT(GAMMAT*UGC*TTH/H2MMT)

**    Calculate exit mach number   **
```

```
      AMACH=3.00
 50   ANSW=(2*(1+((GAMMAC-1)/2)*AMACH**2)/(GAMMAC+1))
     &**((GAMMAC+1)/(GAMMAC-1))
      ANSW=ANSW/AMACH**2-9E4
      if(ANSW.gt.0) goto 60
      AMACH=AMACH+.05
      goto 50

**    Calculate exit conditions   **

 60   VE=SPIMP*GRAV
      TE=H2MMC*(VE/AMACH)**2/(UGC*GAMMAC)
      PE=PRES/((1+((GAMMAC-1)/2)*AMACH**2)**(GAMMAC/
     &(GAMMAC-1)))
      PE=PE/ATM
      AE=300*ATH
      GAMMAE=1.40018-3.60774E-6*TE-2.87154E-8*TE**2+5.90601E-
     &12*TE**3
      GAMMAE=GAMMAE-2.94162E-16*TE**4
      H2MME=-2.8375E-6*TE+2.01588

**    Calculate density of uranium   **

      RHOU=CRITM/VU

**    Calculate diameter of uranium  **

      DIAMU=2*(3*VU/4/PI)**(.3333333)

**    Calculate reactor power      **

      QR=((TU/950)**6.25)*DIAMU/PRES

**    Calculate thickness and mass of pressure shell **

      THPS=PRES*(DIAM+1.52)/(5.516E9)
      PSM=1333.33*PI*((DIAM+1.52+2*THPS)**3-(DIAM+1.52)**3)

**    Calculate mass of moderator, turbopump, nozzle, and
**    radiator  **

      EMM=191.67*PI*((DIAM+1.54)**3-DIAM**3)
      TPM=.000307181*H2DOT*((1.5*PRES)**(.66667))
      ENM=52861.26*THRUST/PRES
      PR=3E-6*SPIMP+4.4E-2
      RM=145*PR*QR

**    Calculate total engine mass   **

      ENGMASS=PSM+EMM+TPM+ENM+RM

**    Calculate engine specific mass and jet power  **
```

```
      ALPHA=204.96*ENGMASS/(SPIMP*THRUST)
      PJET=.5*THRUST*SPIMP*GRAV/1E6
**    Calculate exit velocity       **

      VEXIT=SPIMP*GRAV

**    Calculate thrust to weight ratio  **

      TWR=THRUST/(ENGMASS*GRAV)
      PRES=PRES/ATM
      write(1,70)
 70   format(' SPECIFIED VALUES:')
      write(1,80)SPIMP,THRUST,VUVC,DIAM
 80   format(1X,'Isp=',F8.2,' seconds',3X,'Thrust=',F11.2,'
     &Newtons',3X,'Vu/Vc=',F3.2,3X,'Dc=',F5.2,' Meters')
      write(1,90)
 90   format(//,1X,'CALCULATED ENGINE PARAMETERS:')
      write(1,100)
 100  format(/,1X,'ENGINE CAVITY CONDITIONS:')
      write(1,110)TH2,TU
 110  format(1X,'T(H2)=',F8.2,' Kelvin',19X,'T(U233)=',F9.2,'
     &Kelvin')
      write(1,120)PRES,CRITM
 120  format(1X,'Cavity Pressure=',F7.2,' Atm',13X,
     &'Critical Mass U233=',F6.2,' Kg')
      write(1,130)VC,VU
 130  format(1X,'Cavity Volume=',F5.2,' Cubic
     &Meters',8X,'Uranium
     &Volume=',F5.2,' Cubic Meters')
      write(1,140)DIAMU,RHOU
 140  format(1X,'Uranium Diameter=',F4.2,' Meters',12X,'Uranium
     &Density=',F6.2,' Kg/Cubic Meter')
      write(1,150)H2MMC,GAMMAC
 150  format(1X,'Hydrogen Molecular
     &Mass(Cavity)=',F5.3,2X,'Gamma(Cavity)=',F5.3)
      write(1,160)
 160  format(/,1X,'MASS FLOW RATES:')
      write(1,170)DOTM,PDOT
 170  format(1X,'H2-To-U233 Mass Flow Ratio=',F7.2,6X,
     &'Propellant Mass Flow Rate=',F6.2,' Kg/sec')
      write(1,180)H2DOT,UDOT
 180  format(1X,'H2 Mass Flow Rate= ',F7.3,' Kg/sec',6X,' U233
     &Mass Flow Rate=',F6.3,' Kg/sec')
      write(1,190)
 190  format(/,1X,'ENGINE DIMENSIONS:')
      write(1,200)THPS,PSM
 200  format(1X,'Pressure Shell Thickness=',F5.3,'
     &Meters',3X,'Pressure Shell Mass=',f8.2,' Kg')
      write(1,210)EMM
 210  format(1X,'Moderator Thickness= 0.76 Meters'
     &,8X,'Moderator Mass=',F9.2,' Kg')
      write(1,220)TPM,ENM
```

```
220     format(1X,'Turbo Pump Mass=',F7.2,' Kg',14X,'Exhaust
       &Nozzle Mass=',F7.2,' Kg')
        write(1,230)RM,ENGMASS
230     format(1X,'Radiator Mass=',F9.2,' Kg',14X,'Total Engine
       &Mass=',F10.2,' Kg')
        write(1,240)
240     format(/,1X,'VARIOUS ENGINE PARAMETERS:')
        write(1,250)QR,.06*QR
250     format(1X,'Reactor Power=',F8.2,' MWatts',11X,'Radiated
       &Power=',F7.2,' MWatts')
        write(1,260)ALPHA,PJET
260     format(1X,'Engine Specific Mass=',F7.5,' Kg/KWatt',
       &3X,'Jet Power=',F9.2,' MWatts')
        write(1,270)TWR,PJET/QR
270     format(1X,'Thrust to Weight Ratio=',F7.5,10X,'Engine
       &Efficiency=',F5.4)
        write(1,280)
280     format(/,1X,'THROAT CONDITIONS:')
        write(1,290)TTH,PTH
290     format(1X,'Throat temperature=',F8.2,' Kelvin',6X,'Throat
       &pressure=',F7.2,' Atm')
        write(1,295)H2MMT,GAMMAT
295     format(1X,'Hydrogen Molecular Mass
       &(Throat)=',F5.3,2X,'Gamma (Throat)=',F5.3)
        write(1,300)ATH,VTH
300     format(1X,'Throat Area=',F7.4,' Sq Meters',11X,'Throat
       &Velocity=',F9.2,' Meters/sec')
        write(1,310)
310     format(/,1X,'EXIT CONDITIONS:')
        write(1,320)TE,PE
320     format(1X,'Exit temperature=',F8.2,' Kelvin',8X,'Exit
       &pressure=',f5.3,' Atm')
        write(1,325)H2MME,GAMMAE
325     format(1X,'Hydrogen Molecular Mass(Exit)=',
       &F5.3,4X,'Gamma(Exit)=',F5.3)
        write(1,330)AE,VE
330     format(1X,'Exit Area=',F7.4,' Sq Meters',13X,'Exit
       &Velocity=',F9.2,' Meters/sec')
        write(1,340)AMACH
340     format(1X,'Exit-To-Throat Area Ratio= 300',10X,'Exit Mach
       &Number= ',F5.2)
*************************
* This part of the program uses the method of Masser from Chapter
* 5 to find the constant of integration for the calculation of
* the dose obtained from the plume
*
        f(1)=.07
        f(2)=.00002
        do i=1,2
        BIG=GAMMAE*SQRT(2/(GAMMAE-1)*(2/(GAMMAE+1))**
       &((GAMMAE+1)/(GAMMAE-1)))
        CFCFMAX=(BIG*SQRT(1-(PE/PRES)**((GAMMAE-1)/GAMMAE))+
```

```
     &300*PE/PRES)/BIG
      X=1/(SQRT(PI)*(1-CFCFMAX))
      B=X/4/SQRT(PI)*((GAMMAE-1)/(GAMMAE+1))**.5*(2/(GAMMAE+
     &1))**(1/(GAMMAE-1))
      TOP=f(i)*QR*6.448E+7*(H2MME*PE/TE/82.06)*AMACH*B*
     &AE/PI*10000
      BOTTOM=H2DOT*3.7*(1+(GAMMAE-1)/2*AMACH**2)**.5*(2/
     &(GAMMAE+1))**((GAMMAE+1)/2/(GAMMAE-1))
      CONST=TOP/BOTTOM
350   format(//1X,'lambda=',F15.9,5x,'constant=',f20.4)
      write(1,350)X,CONST
360   format(1X,'f(t)=',f10.7)
      write(1,360)f(i)
      enddo
      close(unit=1)
      stop
      end
```

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